RETAINING WALLS FOR EARTH

RETAINING-WALLS FOR EARTH.

INCLUDING

THE THEORY OF EARTH-PRESSURE AS DEVELOPED FROM THE ELLIPSE OF STRESS.

WITH

AN APPENDIX PRESENTING THE THEORY OF PROF. WEYRAUCH.

BY

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PREFACE.

The first edition of this work was based upon the theory advanced by Prof. Weyrauch in 1878, but owing to the length of the demonstrations used by him, it was thought advisable to present different and shorter demonstrations in this edition. To show that the new demonstrations give identical results with those obtained by Prof. Weyrauch, his demonstrations have been given in an appendix as they appeared in the first edition.

The new demonstrations are based upon the theory first advanced by Prof. Rankine in 1858. Those readers who are familiar with Rankine's Ellipse of Stress can omit pages 27 to 35, inclusive, in following the demonstrations.

An attempt has been made to present the theory in a shape easily followed by those who have only a knowledge of algebra, geometry, and trigonometry; whenever calculus has been resorted to, the work has been simplified as much as possible. For convenience in practice, the formulas have been arranged in a condensed shape in Part I, and are followed by numerous examples illustrating their application.

The values of various coefficients have been computed and tabulated and will be found to very materially decrease the labor of substitution in the formulas.

It is hoped that the introduction of a brief treatment of the supporting power of earth in the case of foundations, as well as the formula for determining the breadth of the base of a retaining-wall, will prove acceptable.

For valuable help in the verification of proofs of formulas, and the critical reading of the whole text, I acknowledge the kind assistance of Prof. Thos. Gray.

M. A. H.

TERRE HAUTE, IND., March, 1891.

NOMENCLATURE.

ϕ = the angle of repose, or the maximum angle which any force acting upon any plane within the mass of earth can make with the normal to the plane.

ϵ = the angle made by the surface of the earth with the horizontal; ϵ is *positive* when measured *above* and *negative* when measured *below* the horizontal.

α = the angle which the back of the wall makes with the vertical passing through the heel of the wall; α is *positive* when measured on the *left* and *negative* when measured on the *right* of the vertical.

δ = the angle which the direction of the resultant earth-pressure makes with the horizontal.

ϕ' = the angle of friction between the wall and its foundation.

ϕ'' = the angle of friction between the back of the wall and the earth.

H = the vertical height of the wall in feet.

h = the depth of earth in feet which is equivalent to a given load placed upon the surface of the earth.

B' = the width in feet of the top of the wall.

B = the width in feet of the base of the wall.

Q = the distance in feet from the toe of the wall to the point where R cuts the base.

P = the resultant earth-pressure in pounds against a vertical wall.

E = the resultant earth-pressure in pounds against any wall.

R = the resultant pressure in pounds on the base of the wall.

G = the total weight in pounds of material in the wall.

γ = the weight in pounds of a cubic foot of earth.

W = the weight in pounds of a cubic foot of wall.

p = the intensity of the pressure in pounds on the base of the wall at the toe.

p' = the intensity of the pressure in pounds on the base of the wall at the heel.

p_0 = the average intensity of the pressure in pounds on the base of the wall.

$x = H \tan \alpha$.

RETAINING-WALLS FOR EARTH.

FORMULAS FOR EARTH-PRESSURE.

In the following formulas α and ϵ are considered as *positive*, and the wall is assumed to be one foot long.

Case I. *General case of inclined earth-surface and inclined back of wall.*

$$E = \frac{H^2\gamma}{2}\frac{\cos(\epsilon-\alpha)}{\cos^2\alpha\cos\epsilon}\times$$

$$\sqrt{\begin{array}{l}\sin^2\alpha+\cos^2(\epsilon-\alpha)\left\{\dfrac{\cos\epsilon-\sqrt{\cos^2\epsilon-\cos^2\phi}}{\cos\epsilon+\sqrt{\cos^2\epsilon-\cos^2\phi}}\right\}^2\\ +2\sin\epsilon\sin\alpha\cos(\epsilon-\alpha)\left\{\dfrac{\cos\epsilon-\sqrt{\cos^2\epsilon-\cos^2\phi}}{\cos\epsilon+\sqrt{\cos^2\epsilon-\cos^2\phi}}\right\}\end{array}}\ ; \quad (1)$$

or

$$E = \frac{H^2\gamma}{2}(B)\sqrt{(C)+(D)A^2+(E)A}. \quad . \quad (1')$$

$$\tan\delta = \frac{\sin\alpha\cos\epsilon+\sin\epsilon\cos(\epsilon-\alpha)A}{\cos\epsilon\cos(\epsilon-\alpha)A}; \quad . \quad (1a)$$

or

$$\tan\delta = \frac{\sin\alpha}{\cos(\epsilon-\alpha)A}+\tan\epsilon, \quad . \quad . \quad . \quad . \quad . \quad (1'a)$$

where

$$A = \cos\epsilon \frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}} \quad . \quad . \quad . \qquad (d)$$

CASE II. *Surface of earth inclined and* $\alpha = 0$.

$$E = P = \frac{H^2\gamma}{2}\left\{\cos\epsilon \frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}} = A\right\}. \qquad (2)$$

From Diagram I the values of A can be found for all values of ϕ from 0° to 90° and of ϵ from 0° to 90°, varying by 5°.

$$\delta = \epsilon; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2a)$$

or for all vertical walls the direction of the earth-pressure is parallel to the surface of the earth.

CASE III. *The surface of the earth parallel to the surface of repose.*

$$\epsilon = \phi.$$

$$E = \frac{H^2\gamma}{2} \frac{\cos(\phi - \alpha)}{\cos^2\alpha \cos\phi} \sqrt{\frac{\sin^2\alpha + \cos^2(\phi - \alpha)}{+ 2\sin\alpha \sin\phi \cos(\phi - \alpha)}}. \qquad (3)$$

$$\tan\delta = \frac{\sin\alpha + \sin\phi\cos(\phi - \alpha)}{\cos\phi\cos(\phi - \alpha)}. \quad . \quad . \quad . \qquad (3a)$$

CASE IV. *The surface of the earth parallel to the surface of repose and the back of the wall vertical.*

$$\epsilon = \phi \quad \text{and} \quad \alpha = 0.$$

$$E = \frac{H^2\gamma}{2} \cos\phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

$$\delta = \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4a)$$

CASE V. *The surface of the earth horizontal.*

$$\epsilon = 0.$$

$$E = \frac{H^2\gamma}{2}\sqrt{\tan^2 \alpha + \tan^4\left(45^\circ - \frac{\phi}{2}\right)}. \quad . \quad (5)$$

$$\tan \delta = \frac{\tan \alpha}{\tan^2\left(45^\circ - \frac{\phi}{2}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

CASE VI. *The surface of the earth horizontal and the back of the wall vertical.*

$$\epsilon = 0 \quad \text{and} \quad \alpha = 0.$$

$$E = \frac{H^2\gamma}{2}\tan^2\left(45^\circ - \frac{\phi}{2}\right) \quad . \quad . \quad . \quad . \quad (6)$$

$$\delta = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

CASE VII. *Fluid pressure.*

$$\epsilon = \phi = 0.$$

$$E = \frac{H^2\gamma}{2\cos\alpha}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$\delta = \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

GRAPHICAL CONSTRUCTIONS FOR DETERMINING THE THRUST OF EARTH.

The following constructions are perfectly general, and apply to *any plane* within a mass of earth. When applied

for determining the thrust of earth against a *retaining-wall*, α and ϵ are taken as *positive*.

* *Construction* (*a*).

Let BE represent the surface of the earth and BA the back of the wall. Draw AF parallel to BE, and at any point D in AF lay off DF equal to the vertical DE. Draw

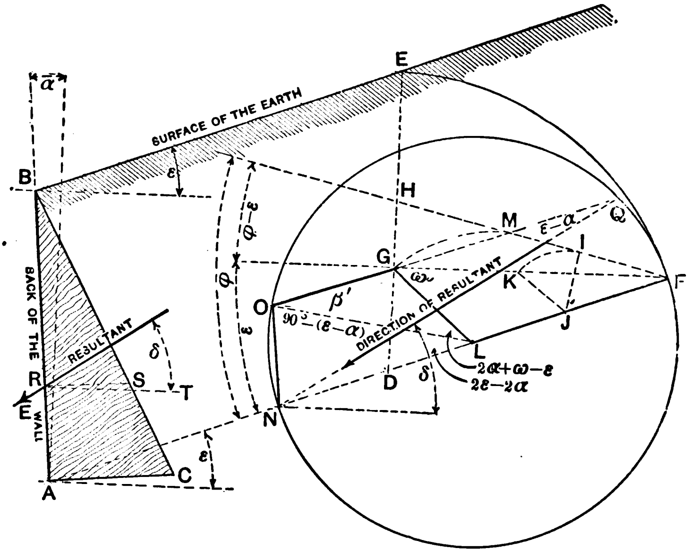

FIG. 1.

FG horizontal, and FH, making the angle ϕ with DF. With any point J in DF describe the arc KI tangent to HF at I cutting FG at K, and draw GL parallel to KJ; with L as a centre and LF as radius, describe the circumference $FQON$ cutting AD at N. Through N draw NO

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* See "Theorie des Erddruckes auf Grund der neueren Anschauungen," by Prof. Weyrauch, 1881.

parallel to AB cutting the circumference $FQON$ at O; at A draw AC equal to OG and normal to AB; the area of the triangle ABC multiplied by γ will be the thrust of the earth on the wall.

To determine the direction of the thrust E, prolong OG to Q; then QN will be the direction of the thrust.

This thrust acts on the wall at $\frac{2}{3}AB$ below B.

* *Construction* (*b*).

Let BQ represent the surface of the earth, and BA the back of the wall. Draw AD parallel to BQ, and at any

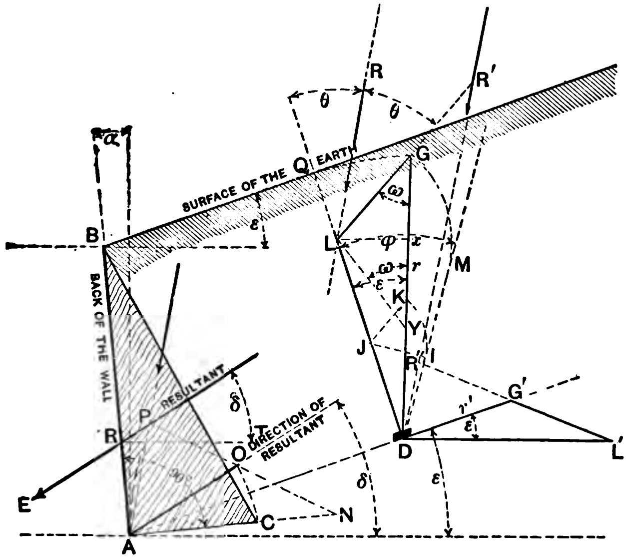

Fig. 2.

point D in AD draw the vertical DG equal to the normal DQ; draw DM making the angle ϕ with the normal DQ.

* This construction follows directly from Rankine's Ellipse of Stress. See Rankine's Applied Mechanics.

At any point J in DQ as a centre, describe the arc IK tangent to DM cutting DG at K, and draw GL parallel to JK. Bisect the angle QLG, and at A draw AP parallel to LR. At A draw AN normal to AB and equal to DL; with N as a centre and AN as radius, describe an arc AP cutting AP at P; connect P and N, and make NO equal to LG; with A as a centre and AO as a radius, describe the arc OC cutting AN at C; then the area of the triangle ABC multiplied by γ will be the thrust against the wall. The direction of this thrust is parallel to AO and it is applied at $\frac{2}{3}AB$ below B.

The constructions (a) and (b) give identical results in every case.

TRAPEZOIDAL AND TRIANGULAR WALLS.

Formulas for the width of the base of trapezoidal walls under the condition that the resultant R cuts the base at a point distant from the toe of the wall equal to one third the width of the base, or $Q = \frac{1}{3}B$.

CASE I. *The general case in which the back of the wall is inclined, and E makes an angle with the horizontal.*

$$B^2 + B\left(\frac{4E}{HW}\sin\delta + B' - x\right)$$
$$= \frac{2E}{HW}\left(H\cos\delta + x\sin\delta\right) + 2B'x + B'^2. \quad . \qquad (8)$$

CASE II. *The back of the wall vertical.*

$$x = 0.$$

$$B^2 + B\left(\frac{4E}{HW}\sin\delta + B'\right) = \frac{2E}{W}\cos\delta + B'^2. \qquad (9)$$

CASE III. *The back of the wall vertical and the thrust normal to the wall.*

$$x = 0 \quad \text{and} \quad \delta = 0.$$

$$B^2 + BB' = \frac{2E}{W} + B'^2. \quad . \quad . \quad . \quad . \quad (10)$$

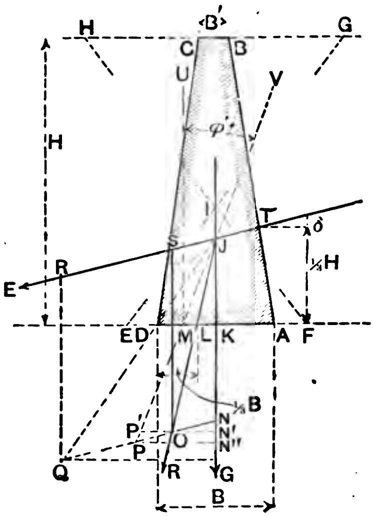

FIG. 3.

If $B = B'$ and $x = 0$, the section of the wall is a rectangle, and (9) becomes

$$B^2 + B\frac{4E}{HW}\sin\delta = \frac{2E}{W}\cos\delta, \quad . \quad . \quad . \quad (9a)$$

and (10) becomes

$$B = \sqrt{\frac{2E}{W}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10a)$$

Formulas for the width of the base of triangular walls under the condition that the resultant R cuts the base at a point distant from the toe of the wall equal to one third the width of the base, or $Q = \frac{1}{3}B$.

CASE I. *The general case in which the back of the wall is inclined, and E makes an angle with the horizontal.*

$$B^2 + B\left(\frac{4E}{HW}\sin\delta - x\right) = \frac{2E}{HW}(H\cos\delta + x\sin\delta). \quad (11)$$

CASE II. *The back of the wall vertical.*

$$\alpha = 0.$$

$$B^2 + B\left(\frac{4E}{HW}\sin\delta\right) = \frac{2E}{W}\cos\delta. \quad . \quad . \quad (12)$$

CASE III. *The back of the wall vertical, and the thrust normal to the wall.*

$$x = 0 \quad \text{and} \quad \delta = 0.$$

$$B = \sqrt{\frac{2E}{W}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The above formulas do not contain the condition that R shall not make an angle greater than ϕ' with the normal to the base of the wall.

From Fig. 3,

$$\tan\phi' \geqq \frac{E\cos\delta}{G + E\sin\delta} = \tan LJK, \quad . \quad . \quad (14)$$

which expresses the condition under which the wall will not slide,

DEPTH OF FOUNDATIONS.

CASE I. *When the intensity of the pressure on the earth is uniform.*

Letting x' equal the depth of the foundation below the surface,

$$x' = \frac{p_0(1 - \sin\phi)^2}{(1 + \sin\phi)^2\gamma - W(1 - \sin\phi)^2}, \quad . \quad . \quad (15)$$

when the weight of the foundation is included; and

$$x' = \left\{\frac{1 - \sin\phi}{1 + \sin\phi}\right\}^2 \frac{p_0}{\gamma}, \quad . \quad . \quad . \quad . \quad (16)$$

when the weight of the foundation is not included.

x' is the minimum depth to which the foundation must be extended for equilibrium. The actual depth should be based upon the minimum value which ϕ is likely to have under any condition of the earth.

CASE II. *When the intensity of the pressure on the earth is uniformly varying.*

$$x' = \frac{p_0}{\gamma} \frac{(1 - \sin\phi)^2}{1 + \sin^2\phi}, \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where x' is the minimum depth to which the foundation must be extended for equilibrium;

$$x_0 = \frac{1}{3} \frac{\sin\phi}{1 + \sin^2\phi}, \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where x_0 is the maximum distance from the centre of the base of the foundation to the point where the resultant pressure cuts the base of the foundation.

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$$P = \frac{(x')^2 \gamma}{2} \frac{1 + \sin \phi}{1 - \sin \phi}, \quad . \quad . \quad . \quad . \quad (21)$$

where P represents the maximum resultant pressure which horizontal earth can resist, when P is applied against a vertical plane of the depth x'.

APPLICATIONS.

The determination of the earth-pressure by the preceding formulas and graphical constructions is a very simple operation when the angle ϕ has been determined or assumed. That care and judgment be used in assuming the value of ϕ is very important, since a change of a few degrees in the value of ϕ sometimes causes a large change in the value of E. An inspection of Diagram I shows that the value of the coefficient A increases very rapidly as ϕ decreases.

When the earth to be retained contains springs, the bank must be thoroughly drained if it is to be retained by an economical tight wall; if it is not drained, the angle ϕ will be likely to become very small as the earth becomes wet.

When the location of the earth to be retained is subjected to jars, the value of ϕ will be decreased.

Hence, in assuming the value of ϕ, the engineer must be sure that the value assumed will be the least value which, in his judgment, it is likely to have.

In constructing the wall the judgment and authority of the engineer must again be exercised in order that the wall be constructed as designed.

In all cases, to insure perfect drainage between the back

of the wall and the earth, numerous "weep-holes" should be provided in the body of the wall, or proper arrangements made to carry away the water at the base of the wall. To facilitate drainage, the backing resting against the wall should be sand or gravel.

In no case should water be permitted to get under the foundation of the wall, neither should the earth in front of the wall be allowed to become wet.

In cold localities the back of the wall near the top should have a large batter to prevent the frost from moving the top courses of stone. As a guard against sliding, the courses of the wall should have very rough beds. The strength of a wall is increased the nearer it approaches a monolith.

Care should be taken to have the foundation broad and deep enough to prevent sliding and upheaving of the earth in front. In clay the foundation should be deep, while in sand or gravel it may be broad and shallow.

The following examples illustrate the application of the formulas:

Ex. 1. Design a trapezoidal wall of sandstone, weighing 150 lbs. per cubic foot, having a width of 3 ft. on top, a height of 30 ft., and the back inclining forward 5°, to retain a bank of sand sloping upward at an angle of 20°.

Data.

$\gamma = 100$ lbs., $W = 150$ lbs.; $\epsilon = 20°$, $\phi = 39°$, $\alpha = 5°$; $H = 30$ ft., $B' = 3$ ft., $x = 2.63$ ft.

1°. *Graphical determination of the values of E and δ.*

The graphical solution of the problem is shown in Fig. 4, where E is found to equal 15,000 pounds. δ lies between 35° and 36°.

2°. *Algebraic determination of E and δ.*

$$E = \frac{H^2\gamma}{2}(B)\sqrt{(C) + (D)A^2 + (E)A} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1')$$

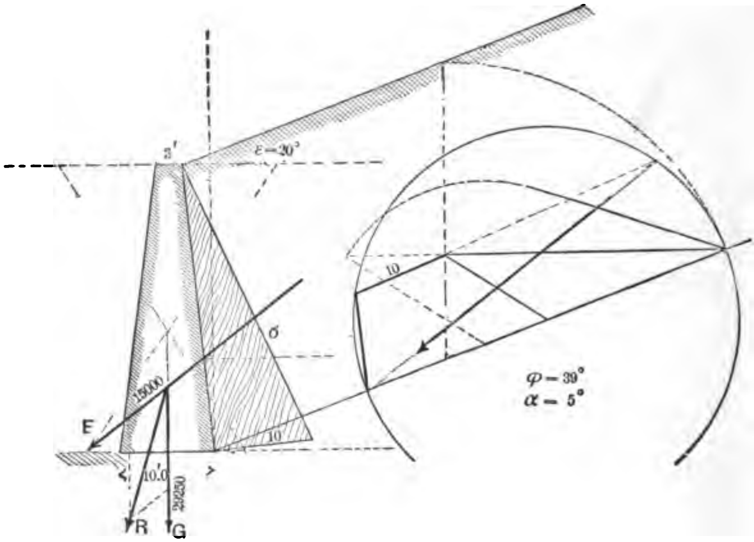

FIG. 4.

Substituting the values of B, C, D, and E as given in the tables, and that of A as given by Diagram I, this becomes

$$E = \frac{900 \times 100}{2}(1.036) \times$$

$$\sqrt{(0.008) + (1.057)(0\,264)^2 + (0.061)0.264},$$

$$E = 45{,}000\ (1.036)\sqrt{0.098} = 14{,}500 \text{ lbs.}$$

$$\tan\delta = \frac{\sin\alpha}{\cos(\epsilon - \alpha)A} + \tan\epsilon, \quad . \quad . \qquad (1'a)$$

$$\tan \delta = \frac{0.087}{0.966(0.264)} + 0.364,$$

$$\tan \delta = 0.705 = \tan 35° \, 11', \text{ about.}$$

3°. *Algebraic determination of the value of B under the assumption that $Q = \frac{1}{3}B$.*

$$B^2 + B \left\{ \frac{4E}{HW} \sin \delta + B' - x \right\}$$

$$= \frac{2E}{HW} \left\{ H \cos \delta + x \sin \delta \right\} + 2B'x + B'^2. \quad . \quad (8)$$

$$B^2 + B \left\{ \frac{4 \times 14500}{30 \times 150} 0.576 + 3 - 2.63 \right\}$$

$$= \frac{2 \times 14500}{30 \times 150} \{30 \times 0.817 + 2.63 \times 0.576\} + 6 \times 2.63 + 9,$$

$$B^2 + 7.79B = 172.53,$$

$$B = -3.89 \pm \sqrt{172.53 + \overline{3.9}^2};$$

$$\therefore B = 13.69 - 3.89 = 9.80 \text{ ft.};$$

or, practically, 10 feet is the required width of the base.

4°. *To determine if the wall will slide on a foundation of sandstone.*

From (14),

$$\tan \phi' \geqq \frac{E \cos \delta}{G + E \sin \delta}.$$

Taking $B = 10$ ft., $G = \frac{10 + 3}{2} 30 \times 150 = 29250$ lbs.

$\delta = 35° \, 11'$, $\cos \delta = 0.817$, and $\sin \delta = 0.576$, then

$$\frac{E \cos \delta}{G + E \sin \delta} = \frac{14500 \times 0.817}{29250 + 14500 \times 0.576} = 0.315.$$

From Table II, the value of $\tan \phi'$ for masonry is 0.6 to 0.7; hence there is no danger of the wall sliding on the foundation.

5°. *To determine the minimum depth to which the foundation must extend consistent with the stability of the earth.*

First determine the maximum value of x_0. From (20),

$$x_0 = \frac{1}{3} \frac{\sin \phi}{1 + \sin^2 \phi},$$

where ϕ must be assumed at its minimum value. Assume that the minimum value of ϕ in this case is 30°; then

$$x_0 = \frac{1}{3} \frac{0.577}{1.333} = 0.133,$$

showing that the resultant must cut the base of the foundation within 0.133 feet of its centre. The resultant cuts the base of the wall 1.67 feet from the centre of its base; hence the width of the foundation must be increased.

Assuming that the depth to which the foundation extends is 4 feet, and that it is vertical in the rear; then the direction of the resultant pressure (not including the additional weight of the foundation) will cut the base of the foundation 7.93 feet from the rear or heel. The required width of the base of the foundation is $(7.93 - 0.13)2 = 15.6$; say, 16 feet.

The value of p_0 can now be found, which corresponds to the assumed value of $x' = 4$ feet.

From (19),

$$p_0 = x'\gamma \frac{1 + \sin^2 \phi}{(1 - \sin \phi)^2};$$

$$p_0 = 400 \frac{1.333}{0.179} = 2960 \text{ lbs.}$$

The average intensity of the pressure on the base of the foundation due to the resultant R is

$$\frac{29250 + 14500 \sin \delta}{16} = 2350 \text{ lbs.}$$

The foundation adds an intensity equal to $4 \times 150 = 600$ pounds approximately; hence the actual value of $p_0 = 2350 + 600 = 2950$ pounds; therefore, if the foundation has a depth of 4 feet and a base of 16 feet, the wall will not sink nor the earth in front of the wall heave, until ϕ becomes less than 30°.

6°. *To determine if the wall and foundation will slide on the earth.*

This is resisted in two ways—by the friction between the masonry and the earth, and by a prism of earth in front of the wall.

The horizontal force tending to make the wall slide equals $E \sin \delta$, or $14500 \cdot 0.576 = 8352$ pounds. The horizontal force tending to make the foundation slide equals the resultant earth-pressure on the rear face of the foundation, which is vertical and 4 feet in height. From (6),

$$E = \left\{ \frac{(30 + 4)^2}{2} - \frac{30^2}{2} \right\} \gamma \tan^2 \left(45° - \frac{\phi}{2}\right),$$

or $\qquad E = 12800 \times 0.226 = 2893.$

Then the total horizontal force tending to make the wall slide is

$$8352 + 2893 = 11245 \text{ lbs.}$$

From Table II the tangent of the angle of friction between masonry and moist clay is 0.33, which evidently is much smaller than the tangent of the actual angle of friction between masonry and dry earth. Assume this tangent to be 0.500.

The total vertical pressure upon the base of the foundation is 37600 pounds, hence the ability to resist sliding is 37600 (0.5) = 18800 pounds, which is much larger than 11245; hence there is no danger of the wall slipping, even if the earth in front of the wall does not act.

Ex. 2. Design a trapezoidal wall of sandstone weighing 150 lbs. per cubic foot, having a width of 3 ft. on top, a height of 30 ft., and the back inclining backward 15°, to retain a bank of sand sloping upward at an angle of 30°.

Data.

$\gamma = 100$ lbs., $W = 150$ lbs.; $\epsilon = 30°$, $\phi = 33°$, $\alpha = -15°$; $H = 30$ ft., $B' = 3$ ft., $x = 8$ ft.

1°. *Graphical determination of the values of E and δ.*

In Fig. 5, let EG represent the surface of the earth, and AB the back of the wall. Draw AF parallel to BG, and from any point D' in AF lay off $D'F$ equal to the vertical $D'G$, and draw FL horizontal; lay off the angle $IFD' = \phi = 33°$, and locate the point M in $D'F$ so that if an arc be described with M as a centre and LM as a radius the arc will be tangent to IF; then with M as a centre and MF as a radius, describe the circumference FHJ and draw JH

parallel to AB; at A draw AL perpendicular to AB and equal to HI. Then

$$\frac{(AB)(AL)}{2}\gamma = \frac{(30.9)(9.6)}{2}100 = 14800 = E.$$

To determine δ, prolong HI to K and draw KJ. Then the angle which this line makes with the horizontal is equal to δ, which is 6° to 7° in this case.

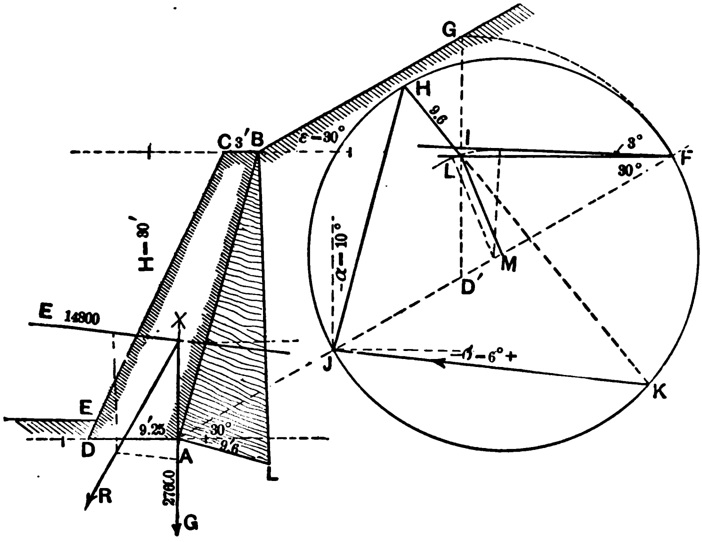

FIG. 5.

2°. *Algebraic determination of E and δ.*
Substituting in (1) and remembering that α is negative,

$$E = 45000\,(0.875)\sqrt{0.067 + 0.183 - 0.111} = 14600 \text{ lbs.}$$

From (1'a),

$$\tan \delta = \frac{-0.259}{0.707(0.524)} + .577 = -0.123 = \tan(-7°).$$

3°. *Algebraic determination of the value of B under the assumption that $Q = \frac{1}{3}B$.*

Substituting the proper values in (11) and remembering that α is negative,

$$B = -4.7 \pm \sqrt{163.44 + (4.7)^2} = 9.0 \text{ ft.}$$

The foundation can be designed in the manner outlined in Ex. 1.

Ex. 3. Determine the dimensions of a brick wall having a vertical back to retain a bank of sand sloping upward at an angle of 20°. $\phi = 30°$, $H = 20'$, $B' = 2'$, $\gamma = 100$.

1°. *Algebraic determination of E and δ.*

Since $\alpha = 0$,

$$E = \frac{H^2\gamma}{2}A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$E = \frac{400 \times 100}{2} 0.424 = 8480; \text{ say, } 8500 \text{ lbs.}$$

The value of A is readily found from Diagram I.

$$\delta = \epsilon = 20°, \quad \text{since} \quad \alpha = 0.$$

2. *Algebraic determination of the value of B under the condition that $Q = \frac{1}{3}B$.*

$$B^2 + B\left\{\frac{4E}{HW}\sin\delta + B'\right\} = \frac{2E}{W}\cos\delta + B'^2. \quad (9)$$

From Table I, $W = 125$ lbs. Then

$$B^2 + B\left\{\frac{4 \times 8500}{20 \times 125} 0.342 + 2\right\} = \frac{2 \times 8500}{125} 0.940 + 4,$$

or

$$B^2 + 6.65B = 131.84.$$

$$B = -3.36 \pm \sqrt{131.84 + \overline{3.36}^2},$$

and

$$B = -3.36 + 11.96 = 7.60 \text{ ft.}$$

Ex. 4. Determine the value of B in Ex. 3 under the assumption that $\epsilon = 0$ (horizontal earth-surface).

$$E = \frac{H^2\gamma}{2}\left\{\tan^2\left(45° - \frac{\phi}{2}\right) = \frac{1 - \sin\phi}{1 + \sin\phi}\right\}, \quad . \quad (6)$$

or $E = 20000\ (0.333) = 6666$, say 6700 lbs.
Since $\alpha = 0$, and $\epsilon = 0$, $\delta = 0$,

$$B^2 + BB' = \frac{2E}{W} + B'^2; \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$B^2 + 2B = 111.2;$$

$$B = -1 \pm \sqrt{111.2 + 1},$$

and

$$B = -1 + 10.59 = 9.6 \text{ ft.}$$

Ex. 5. Determine the value of B in Ex. 3, under the assumption that $\epsilon = \phi = 30°$.

$$E = \frac{H^2\gamma}{2} \cos\phi = 20000\ (0.866) = 17320 \text{ lbs.}$$

From (9),

$$B^2 + B\left\{\frac{4 \times 17320}{20 \times 125} 0.5 + 2\right\} = \frac{2 \times 17320}{125} 0.866 + 4;$$

$$B^2 + 15.86B = 244.05;$$

$$B = -7.93 + \sqrt{244.05 + 7.93^2}.$$

and $\quad B = -7.93 + 17.52 = 9.6$ ft.

Ex. 6. Determine the resultant pressure against the back of a wall when the surface of the earth carries a load equivalent to 5 feet in depth of sand.

$H = 30$ ft., $\alpha = 10°$, $\phi = 30°$, $\epsilon = 0$, and $\gamma = 100$ lbs.

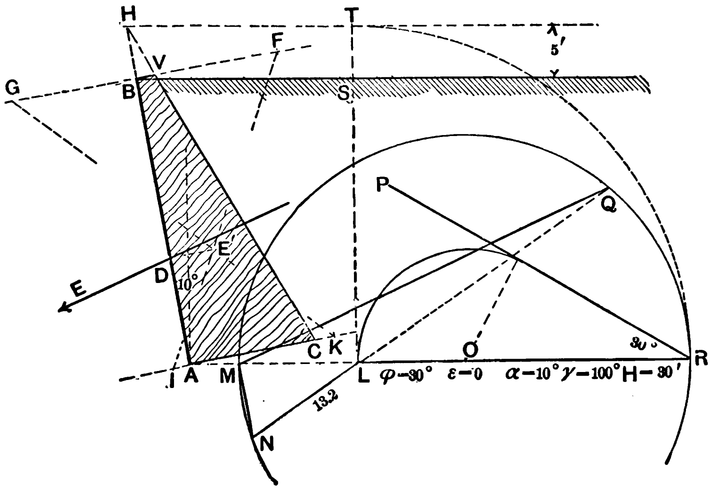

FIG. 6.

Graphical solution of the problem.—In Fig. 6, let BS represent the surface of the earth, and BA the back of the wall.

Make $ST = 5$, and draw HT and BH. Draw AR parallel to BS, parallel to HT, and make LR equal to LT; lay off the angle LRP equal to 30°; with O as a centre

draw an arc passing through L tangent to PR, and then with OR as a radius describe the circumference of the circle RQM, and at M draw MN parallel to AH; at A and normal to AH draw AC equal to NL. Then

$$\frac{AC + BV}{2} BA \,.\, \gamma = E.$$

The direction of E will be parallel to QM.

To determine the point of application of E, find the centre of gravity E' of $ABVC$, and draw $E'D$ parallel to AC, then D will be the point of application of E.

E' can be found as follows: Produce AC and BV, make $AI = CK = BV$, $BG = VF = AC$, and join F and I and G and K. Then E', the intersection of FI and GK, will be the centre of gravity of $ABVC$. BD can be found from the formula

$$BD \cos 10^\circ = \frac{1}{3} \frac{(TL)^3 - 3(TL)(TS)^2 + 2(TS)^3}{(TL)^2 - (TS)^2}.$$

See (30) of Appendix.

Ex. 7. Determine graphically the value of E when $\epsilon = 0$ and $\alpha = 0$, ϕ, γ, and H being given.

In Fig. 7 let BF represent the surface of the earth, and AB the back of the wall. Draw AL parallel to BF and make $IL = IF$; lay off the angle $GLH = \phi$, and at any point K in LH draw MK perpendicular to HL, and lay off $MO = MK$; draw MJ parallel to OI. Then will the arc IN, described with J as a centre and IJ as a radius, pass through I and be tangent to GL; with J as a centre and JL as radius describe the circumference LH; at A lay off $AC = HI$ and normal to AB. Then

$$\frac{AC \times AB}{2} \gamma = E.$$

E is parallel to BF and applied at D, AD being equal to $\frac{1}{3}AB$.

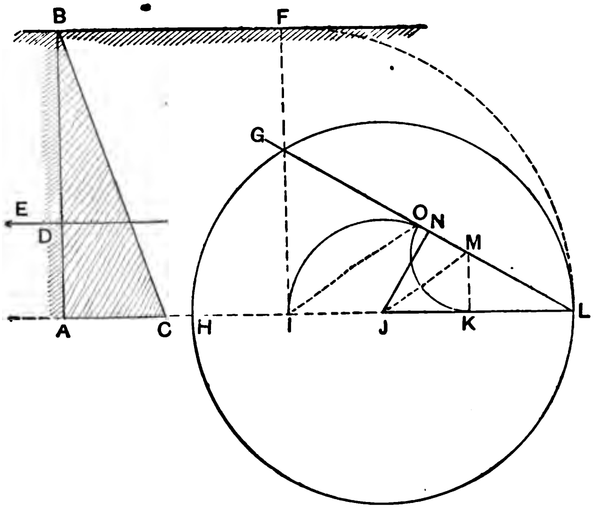

FIG. 7.

Ex. 8. Determine the earth-thrust on the profile shown in Fig. 8, H, γ, ϕ, and ϵ being given.

Graphical solution of the problem.—Let $BCDEA$ represent the given profile, and let the surface of the earth be horizontal. Prolong BC until it intersects SA in S; draw SR normal to BCS and equal to the intensity of the earth-pressure at S; connect B and R. Then from the middle point of BC draw GF parallel to SR; the distance GF multiplied by γ will be the average intensity of the earth-pressure on BC. In a similar manner the average intensities on CD, DE, and EA can be found, and hence the total pressures on each determined. The points of application of these resultant pressures, E_1, E_2, E_3, and E_4,

can be found by the method used in Ex. 6 for finding the centre of gravity of a trapezoid. The directions of

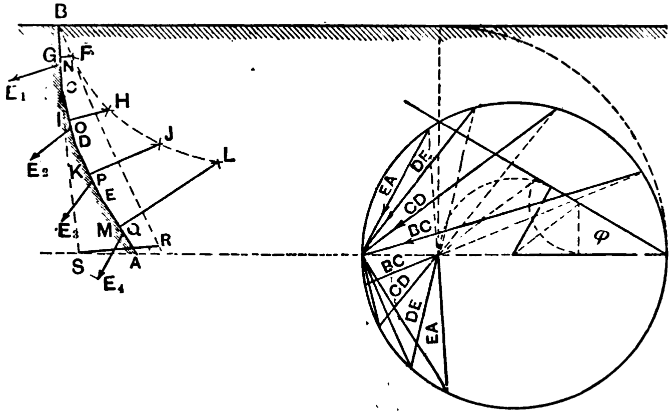

FIG. 8.

E_1, E_2, E_3, and E_4 are found from the construction on the right.

Ex. 9. Determine the thrust of the earth against a vertical wall when ϵ is negative.

For the explanation of this construction, see Part II, page 47, Fig. 8*a*.

Ex. 10. From the following data determine E, δ, and Q:

$$\epsilon = 0,\ \phi = 38°,\ \alpha = 10° 23';\ \gamma = 90 \text{ lbs.},\ W = 170 \text{ lbs.};$$

$$H = 15 \text{ ft.},\quad B = 6 \text{ ft.},\quad B' = 2 \text{ ft.}$$

Ans. $E = 3037$ lbs., $\delta = 27° 13'$, $Q = 2.2$ ft.

Ex. 11. Determine the dimensions of a trapezoidal wall built of dry, rough granite, having a vertical back and being 20 feet high, to safely retain the side of a sand cut,

the surface of the sand being level with the top of the wall. $W = 165$ lbs., $\gamma = 100$ lbs., $\phi = 33°\ 40'$, $H = 20$ ft., $B' = 2$ ft.

Ans. $E = 5734$ lbs., $\delta = 0$, $B = 8$ ft., and $Q = 2.8$ ft., about.

Ex. 12. The same as Ex. 11, with $\alpha = 8°$ instead of $\alpha = 0$.

Ans. $E = 6330$ lbs., $B = 8$ ft., and $Q = 2.7$ ft.

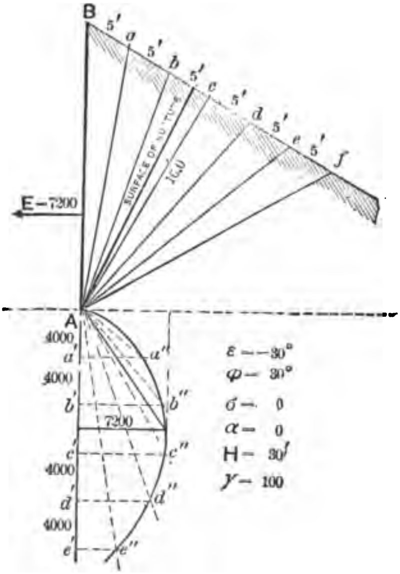

FIG. 8a.

Ex. 13. What must be the dimensions of a rubble wall of large blocks of limestone, laid dry, to retain a sand filling which supports two lines of standard-gauge railroad track? (Assume the depth of sand to produce a pressure on the earth equal to that produced by the railroad and trains as 4 feet.)

$H = 15$ ft., $\alpha = 8°$, $\phi = 33° \ 40'$, $\gamma = 100$ lbs., $W = 170$ lbs., $B' = 3.5$ ft.

Ans. $E = 5760$ lbs., $\delta = 18° \ 7'$, $B = 8$ ft., $Q = 2.7$ ft.

Ex. 14. Determine E, δ, B, and Q, when $W = 170$ lbs., $\gamma = 100$ lbs., $\alpha = 8°$, $\epsilon = \phi = 33° \ 40'$, $H = 20$ ft., $B' = 2$ ft.

Ans. $E = 21760$ lbs., $\delta = 32° \ 25'$, $B = 9$ ft., $Q = 3$ ft.

* Ex. 15. A wall 9 ft. high faces the steepest declivity of earth at a slope of 20° to the horizon; weight of earth 130 lbs. per cubic foot, angle of repose 30°. Determine E when $\alpha = 0$.

Ans. $E = 2187$ lbs.

* Ex. 16. $\epsilon = 33° \ 42'$, $\phi = 36°$, $H = 3$ ft., $\gamma = 120$ lbs., $\alpha = 0$. Determine E.

Ans. $E = 278$ lbs.

* Ex. 17. $\phi = 25°$, $\epsilon = 0$, $\alpha = 0$, $H = 4$ ft., $\gamma = 120$ lbs., $E = ?$

Ans. $E = 390$ lbs.

* Ex. 18. $\phi = 38°$, $\epsilon = 0$, $\alpha = 0$, $H = 3$ ft., $\gamma = 94$ lbs., $E = ?$

Ans. $E = 100.5$ lbs.

* Ex. 19. A ditch 6 feet deep is cut with vertical faces in clay. These are shored up with boards, a strut being put across from board to board 2 feet from bottom, at intervals of 5 feet apart. The coefficient of friction of the moist clay is 0.287, and its weight 120 lbs. per cubic foot. Find the thrust on a strut, also find the greatest thrust which might be put upon the struts before the adjoining earth would heave up.

Ans. $E =$ 1230 lbs.
Thrust per strut = 6128 lbs.
Greatest thrust = 19029 lbs.

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* Alexander's Applied Mechanics.

* Ex. 20. A wall 10 ft. high, 2 ft. thick, and weighing 144 lbs. per cubic ft., is founded in earth weighing 112 lbs. per cubic ft., and whose angle of repose is 32°. Find the least depth of the foundation.

Ans. $x' = 1.21$ ft. $10 - 1.21 = 8.79$ ft. = amount of wall above the ground.

* Ex. 21. An iron column is to bear a weight of 20 tons (2240 lbs. = one ton); the foundation is a stone 3 ft. square on bed, sunk in earth weighing 120 lbs. per cu. ft.; angle of repose 27°. Find the least depth to which it must be sunk for equilibrium.

Ans. $x' = 6$ ft.

* Ex. 22. A brick wall, allowing for openings, weighs 42000 lbs. per rood of 36 sq. ft. (on an average one brick and a half), and stands 45 ft. above the ground; the foundation is to widen to four bricks at the bottom. Find depth of foundation in clay weighing 130 lbs. per cu. ft. (angle of repose 27°), neglecting weight of unknown foundation.

Ans. $x' = 1.7$ ft.

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* Alexander's Applied Mechanics.

PART II.

THE THEORY OF EARTH-PRESSURE AND THE STABILITY OF RETAINING-WALLS.

Preliminary Principles.—Before demonstrating the general formula for the thrust of earth against a wall, it will be necessary to establish the relations between the stresses in an unconfined and homogeneous granular mass.

* In Fig. 1 let ABC be any small prism within a granu-

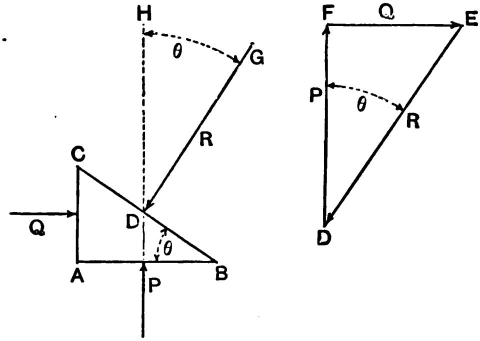

FIG. 1.

lar mass which is in equilibrium un er the action of the three stresses P, Q, and R, having the intensities p, q, and r respectively.

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* In all the demonstrations which follow, the dimension perpendicular to the page will be considered as unity.

Let θ represent the angle of inclination of the plane CB with AB, and the angle at A be a right angle.

The planes AB and AC are called planes of principal stress, and P and Q are called principal stresses.

CASE I. *If the principal stresses are of the same kind and their intensities the same, then will the resultant stress on any third plane be normal to that plane and its intensity be equal to that of either principal stress.*

In Fig. 1, for convenience, let $AB = 1$, then $AC = \tan\theta$, and $CB = \frac{1}{\cos\theta}$. Hence

$$P = p,\ \ Q = q\tan\theta = p\tan\theta, \text{ since } p = q, \text{ and } R = \frac{r}{\cos\theta}.$$

Since P, Q, and R are in equilibrium, they will form a closed triangle, as shown on the right in Fig. 1. Hence

$$R^2 = P^2 + Q^2,$$

or

$$\frac{r^2}{\cos^2\theta} = p^2 + p^2\tan^2\theta = p^2(1 + \tan^2\theta);$$

$$\therefore r = p = q.$$

Also,

$$R\cos FDE = P,$$

or

$$\frac{r}{\cos\theta}\cos FDE = p; \quad \text{but } r = p.$$

Hence

$$\cos\theta = \cos FDE = \cos HDG;$$

$$\therefore HDG = \theta \quad \text{and} \quad R \text{ is normal to } CB.$$

CASE II. *If the principal stresses are not of the same kind but their intensities the same, then will the resultant make the angle θ with the direction of the principal stress, but on the opposite side from that on which the resultant in Case* I *lies, and its intensity be equal to that of either principal stress.*

The demonstration of Case I proves this principle if Fig. 1 is replaced by Fig. 2.

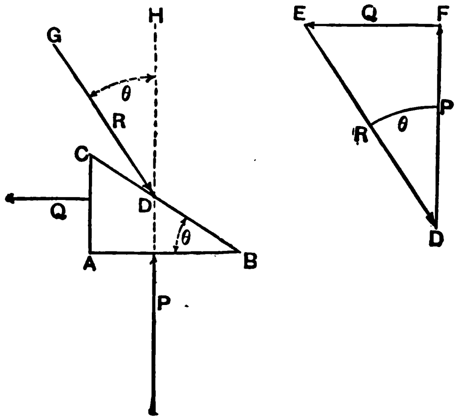

FIG. 2.

CASE III. *Given the principal stresses of the same kind but having unequal intensities, to determine the intensity and direction of the resultant stress on any third plane.*

Let P and Q be compressive and the intensity $p >$ the intensity q.

The following identities can be written:

$$p = \tfrac{1}{2}(p + q) + \tfrac{1}{2}(p - q),$$

and

$$q = \tfrac{1}{2}(p + q) - \tfrac{1}{2}(p - q);$$

or the resultant intensity on the plane *CB* may be considered as being the resultant of two intensities, one being the intensity of the resultant stress caused by two like principal stresses having the same intensity $\frac{1}{2}(p+q)$, and the other the intensity of the resultant stress caused by two unlike principal stresses having the same intensity $\frac{1}{2}(p-q)$.

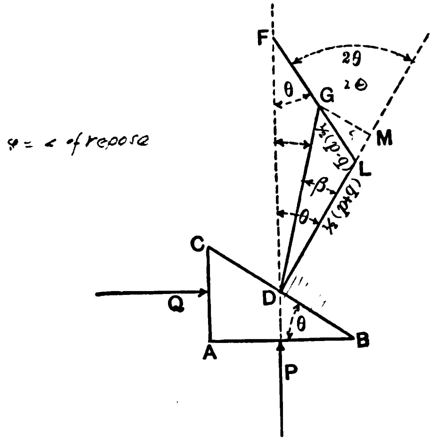

FIG. 3.

The intensity of the resultant stress caused by the first two principal stresses will be, by Case I, $\frac{1}{2}(p+q)$, and the direction of the resultant will be normal to the plane *CB*. By Case II the resultant of the second pair of principal stresses will make the angle θ with the direction of *P*, and its intensity will be $\frac{1}{2}(p-q)$; then the resultant intensity can be found as follows:

In Fig. 3 draw *MD* normal to *BC*, and make $LD = \frac{1}{2}(p+q)$; with *L* as a centre and *LD* as radius, describe an arc cutting *FD* at *F*. Then the angle $LFD = LDF = \theta$. Lay off $LG = \frac{1}{2}(p-q)$, and draw *GD*, which is the result-

ant intensity, and the intensity of the resultant stress on CD caused by the two principal stresses P and Q. GD also represents the direction of the resultant stress R.

Since the intensities of the principal stresses remain constant, $\frac{1}{2}(p+q)$ and $\frac{1}{2}(p-q)$ will remain the same for any inclination of the plane CB; hence the intensity r of the resultant depends upon the angle θ when p and q are given.

From Fig. 3,

$$GL \cos 2\theta = LM \quad \text{and} \quad GL \sin 2\theta = GM,$$

$$DM = DL + LM = \tfrac{1}{2}(p+q) + \tfrac{1}{2}(p-q)\cos 2\theta,$$

$$\overline{GD}^2 = r^2 = \overline{GM}^2 + \overline{DM}^2,$$

or

$$r = \sqrt{p^2 \cos^2 \theta + q^2 \sin^2 \theta}, \quad . \quad . \quad . \quad . \quad (a)$$

which is the general expression for the intensity of the resultant stress of a pair of principal stresses.

As the angle θ changes, the angle β will also change, and it will have its maximum value when the angle $LGD = 90°$. This is easily proven as follows:

With L as centre and GL as radius describe an arc; then β will have its maximum value when the line DG is tangent to the arc; but when DG is tangent to the arc the angle LGD is a right angle, since LG is the radius of the arc.

$$\sin \max \beta = \frac{p-q}{p+q}, \quad . \quad . \quad . \quad . \quad . \quad (b)$$

from which the following can be easily obtained:

$$\frac{p}{q} = \frac{1 - \sin \max \beta}{1 + \sin \max \beta}, \quad . \quad . \quad . \quad . \quad . \quad (c)$$

which expresses the limiting ratio of the intensities of the principal stresses consistent with equilibrium, p being greater than q.

CASE IV. *Given the intensity and direction of the resultant stress on any plane, and the value of max β, to determine the intensities and directions of the principal stresses.*

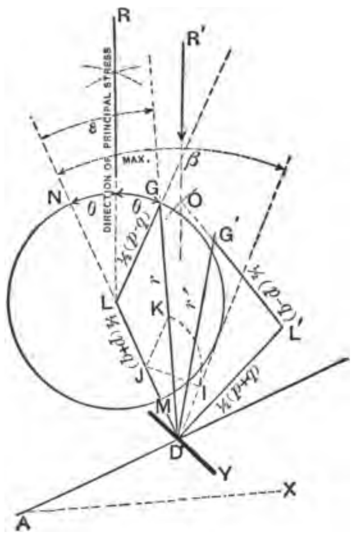

FIG. 4.

Let AD represent the given plane and GD the direction and intensity of the resultant stress at the point D.

Draw DL normal to AD, and draw DI, making the angle max β with LD. At any point J in DL describe an arc tangent to DI, cutting GD in K and draw GL parallel to KJ; with L as a centre and LG as radius describe

a circumference. This circumference will pass through G and be tangent to DI; hence $\frac{GL}{DL} = \sin \max \beta$.

Since $\sin \max \beta = \frac{p-q}{p+q}$, and GL and LD are components of r,

$$GL = \tfrac{1}{2}(p-q) \quad \text{and} \quad DL = \tfrac{1}{2}(p+q);$$

then $ND = NL + LD = \frac{1}{2}(p-q) + \frac{1}{2}(p+q) = p$,

and $MD = LD - LM = \frac{1}{2}(p+q) - \frac{1}{2}(p-q) = q$,

which completely determines the intensities of the principal stresses.

According to Case III, the direction of the greater principal stress bisects the angle between the prolongation of LM and the line GL; hence RL represents the direction of the greater principal stress, and that of the other is at right angles to RL.

The above intensities and directions being determined, the intensity of the resultant stress on any other plane passing through D is easily determined as follows:

Let DY represent any plane passing through D, draw DL' normal to MY and equal to $\frac{1}{2}(p+q)$. Draw $R'D$ parallel to RL, and with L' as a centre and $L'D$ as radius describe an arc cutting $R'D$ at O, and make $L'G' = \frac{1}{2}(p-q)$; then $G'D = r' =$ the intensity of the resultant stress on DY.

It is clear that if the value of max β can be obtained for a mass of earth that the construction of Fig. 3 can be employed in determining the intensity of the earth-pressure at any point in *any plane* within the mass.

It has been established by experiment that if a body be placed upon a plane, that (as the plane is made to incline to the horizontal) at some angle of inclination the body will commence to slide down the plane, and that this angle depends largely upon the *character* of the surfaces in contact.

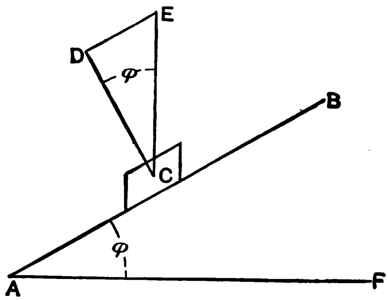

FIG. 5.

In Fig. 5 let AB represent a plane inclined at the angle ϕ with the horizontal, and C any mass just on the point of sliding down the plane. Let EC represent the weight of the mass C, and ED and DC the components respectively parallel and normal to the plane AB. Then DE is the force required to just keep the mass C from sliding down the plane, assuming the plane to be perfectly smooth, or if the plane is rough this force represents the effect of friction.

$$\frac{DE}{DC} = \tan \phi,$$

or when the mass C is about to slide, the resultant pressure EC on AB makes the angle ϕ with the normal to the

plane, the angle ϕ being the inclination of the plane AB, and is called the angle of friction.

In the case of earth, considered as a dry granular mass, the inclination of the steepest plane upon which earth will not slide is called the angle of repose, and the plane the surface of repose.

From the above, then, it follows that in a mass of earth the resultant pressure on any plane cannot make an angle with the normal to that plane which is greater than the angle of repose ϕ; therefore the construction of Case IV applies to earth when max β is replaced by ϕ. The values of ϕ for earth under various conditions are given in Table II.

The preceding principles will now be applied in determining the thrust of earth against a retaining-wall.

EARTH-PRESSURE.

In order that the formulas may not become too complex for practical use, it will be assumed that the earth is a homogeneous granular mass without cohesion. The surface of the earth will be considered to be a plane, and the length of the mass measured normally to the page as unity.

* *Given the intensity and direction of the resultant stress at any point in any plane parallel to the surface of the earth, the inclination of the surface of the earth with the horizontal, and the angle of repose, to determine the intensity and direction of the resultant stress on a vertical plane passing through the same point.*

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* For comparison, see the "Technic," 1888; a construction by Prof. Greene.

The construction follows (see Fig. 4, above) directly from Rankine's Ellipse of Stress.

In Fig. 6 let BQ represent the surface of the earth, and D any point in the plane AD parallel to BQ; draw DQ normal to AD, and make the vertical GD equal to QD; then $GD \cdot \gamma$ is the intensity of the resultant pressure at D. Draw DM, making the angle ϕ with LD, and with L as centre describe an arc tangent to DM and passing through G; then by Case IV $LG \cdot \gamma = \frac{1}{2}(p - q)$, $LD \cdot \gamma = \frac{1}{2}(p + q)$,

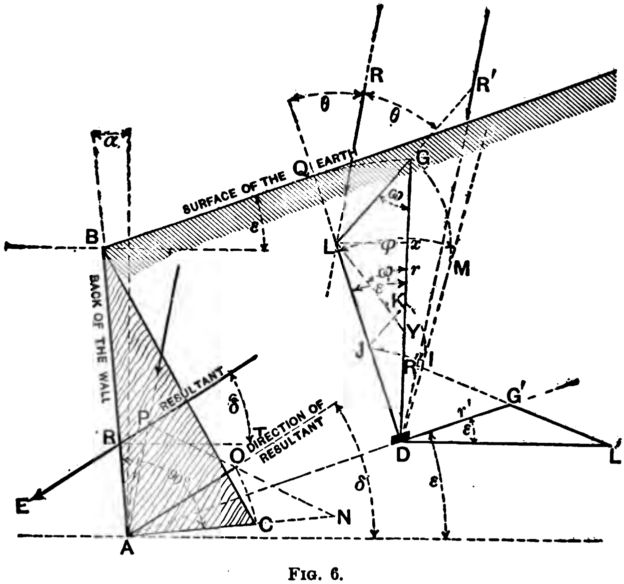

FIG. 6.

and RL bisecting the angle QLG is the direction of the greater principal stress. To determine the intensity and direction of the resultant stress at D on a vertical plane, proceed according to Case IV. Draw $R'D$ parallel to RL and $DL' = DL$ normal to DG. With L' as a centre and $L'D$ as radius describe an arc cutting $R'D$ at R'', and make

$L'G' = LG$; then DG' represents the direction of the resultant stress, and $DG' \cdot \gamma$ the intensity of the resultant.

In Fig. 6 the angle $R'DL' = DR''L' = 90° - \omega + \theta'$. $\therefore G'L'D = 2\omega - 2\theta'$. But $2\theta' = \omega + \epsilon$; hence $G'L'D = \omega - \epsilon$.

Draw $LY = LG$; then the angle $DLY = \omega - \epsilon$. $\therefore$ Since $LD = DL'$ and $LY = LG = L'G'$, the triangle $G'L'D$ equals the triangle LYD and the angle $G'DL' = \epsilon$; or *the direction of the resultant earth-pressure against a vertical plane is parallel to the surface of the earth.*

From Fig. 6,

$$\tfrac{1}{2}(p - q) \cos \omega = GX \cdot \gamma,$$

$$\tfrac{1}{2}(p - q) \sin \omega = LX \cdot \gamma,$$

$$\tfrac{1}{2}(p + q) \cos \epsilon = DX \cdot \gamma.$$

Now
$$DY = DG' = DG - 2GX,$$

or

$$DG' \cdot \gamma = DG \cdot \gamma - (p - q) \cos \omega$$
$$= \tfrac{1}{2}(p + q) \cos \epsilon - \tfrac{1}{2}(p - q) \cos \omega,$$
$$\tfrac{1}{2}(p + q) : \sin \omega \quad :: \quad \tfrac{1}{2}(p - q) : \sin \epsilon,$$

and

$$\sin \omega = \frac{p + q}{p - q} \sin \epsilon,$$

or

$$\cos \omega = \sqrt{1 - \left(\frac{p + q}{p - q}\right)^2 \sin^2 \epsilon} = \sqrt{\frac{(p - q^2) - (p + q)^2 \sin^2 \epsilon}{(p - q)^2}},$$

and since
$$\tfrac{1}{2}(p + q) \sin \phi = \tfrac{1}{2}(p - q),$$

$$\cos \omega = \frac{1}{\sin \phi} \sqrt{\cos^2 \epsilon - \cos^2 \phi}.$$

Substituting this value for $\cos \omega$ in the equation for $DG' \cdot \gamma$, it becomes

$$DG' \cdot \gamma = \tfrac{1}{2}(p+q)\cos\epsilon - \tfrac{1}{2}(p-q)\frac{1}{\sin\phi}\sqrt{\cos^2\epsilon - \cos^2\phi},$$

or since $$\frac{1}{\sin\phi} = \frac{p+q}{p-q},$$

$$DG' \cdot \gamma = \tfrac{1}{2}(p+q)\{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}\}.$$

In a similar manner,

$$DG \cdot \gamma = \tfrac{1}{2}(p+q)\{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}\},$$

and

$$\frac{DG'}{DG} = \frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}};$$

hence

$$DG' = DG\frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}}.$$

Let $x =$ the *vertical* distance between the two planes BQ and AD, then

$$DG = DQ = x\cos\epsilon.$$

$$\therefore\ DG' \cdot \gamma = (x)\,\gamma\cos\epsilon\frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}},$$

which is the expression for the intensity of the resultant earth-pressure on a vertical plane at any depth x below the surface.

Let

$$* A = \cos\epsilon\frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}}. \quad . \quad . \quad (d)$$

* See Rankine's Applied Mechanics; Alexander's Applied Mechanics; Theories of Winkler and Mohr.

The average intensity of the resultant earth-pressure on a vertical plane of the length x will be

$$\left(\frac{x}{2}\right)\gamma A,$$

and hence the total pressure will be

$$P = \frac{x^2}{2}\gamma A. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (e)$$

Since the intensities of the pressures are uniformly varying from the surface, and increasing as x increases, the application of the resultant thrust will be at a depth of $\frac{2}{3}x$ below the surface.

Considering the earth as an unconfined mass, the above formula is perfectly general and can be applied under all conditions, including the case when ϵ is negative.

The resultant stress on any plane as AB, Fig. 6, can be found by applying the principles of Case IV. Draw PA parallel to RL, make $AN = LD$ and $NO = LG$; then AO represents the direction of the resultant pressure on AB. Make $AC = AO$; then the area of the triangle ABC multiplied by γ is the total pressure on the plane AB, and this pressure is applied at $\frac{2}{3}AB$ below B.

In unconfined earth this construction is perfectly general and applies to *any plane.* It also applies equally well to curved profiles. An example illustrating the application of the method will be given in the *applications.* See pages 22 and 23.

The following graphical construction, Fig. 7, is more convenient than that of Fig. 6.

As before, let BE represent the surface of the earth, and

AD a plane parallel to the surface. At any point D in this plane, draw DE vertical and make $DF = DE$; draw FG horizontal and make the angle $HFD = \phi$.

With L as a centre, describe an arc passing through G and tangent to MF; then with L as a centre and LF as

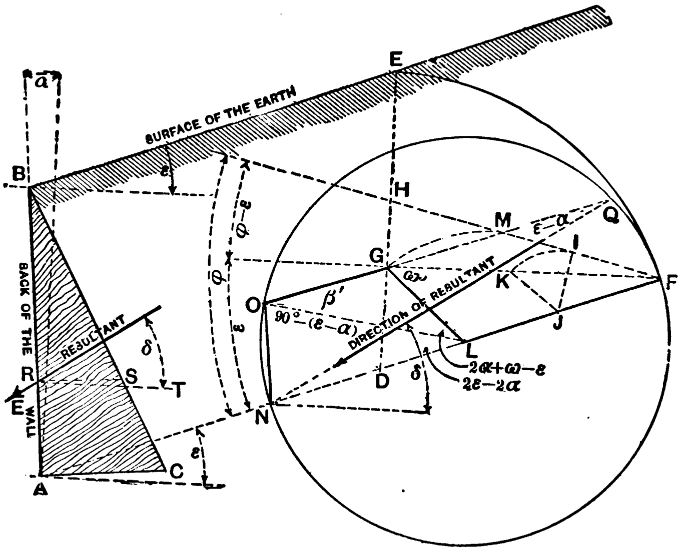

FIG. 7.

radius, describe the circumference FON, cutting AD at N; through N draw NO parallel to AB, then draw AC normal to AB and equal to OG. The area of the triangle ABC multiplied by γ will be the total earth-pressure on AB. To determine the direction of the thrust prolong OG to Q, then QN is the direction of the thrust.

That this construction is equivalent to that of Fig. 6 is

proved as follows. The triangle GLF of Fig. 7 equals the triangle GLD of Fig. 6.

$$\therefore\ GL\cdot\gamma = \tfrac{1}{2}(p-q) \quad \text{and} \quad LF\cdot\gamma = LO\cdot\gamma = \tfrac{1}{2}(p+q).$$

In Fig. 6, the angle $NAP = NPA = 90° - \frac{1}{2}(\omega - \epsilon) - \alpha$.

$$\therefore\ ONA = \omega - \epsilon + 2\alpha.$$

In Fig. 7, the angle $OLN = 2\epsilon - 2\alpha$. But $GLN = \omega + \epsilon$.

$$\therefore\ GLO = \omega - \epsilon + 2\alpha,$$

and GO of Fig. 7 equals AO of Fig. 6.

In Fig. 7, the angle $QNO = 90° - \beta'$.

In Fig. 6, the angle $OAB = 90° - \beta'$.

Therefore the direction of the thrust is the same in both constructions.

The two constructions given above are all that is required to determine the thrust of earth upon any plane within the mass of earth, as one can be used as a check upon the other; but as a formula is often very convenient, a general formula will now be deduced which will enable one to determine the values of E and δ for any plane within a mass of earth.

General Formula for the Thrust of Earth.

In Fig. 8, let BQ represent the surface of the earth and AB any plane upon which the earth-pressure is desired.

Draw AD parallel to BQ and let the vertical distance $QD = FA = x$,

From (e) the earth-pressure upon FA is parallel to the surface and equal to

$$P = \frac{x^2}{2}\gamma A.$$

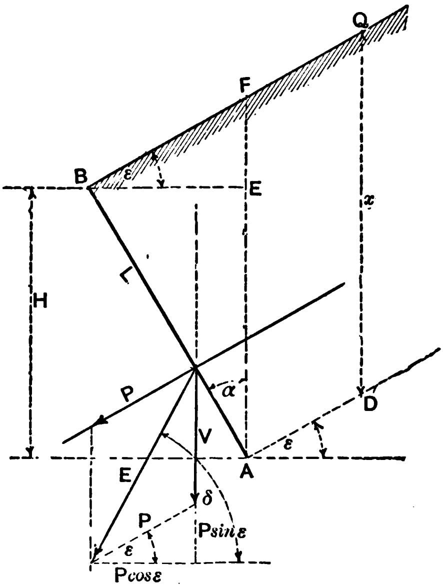

FIG. 8.

But $AF = x = H(1 + \tan\alpha \tan\epsilon) = H\dfrac{\cos(\epsilon - \alpha)}{\cos\alpha \cos\epsilon}$;

$$\therefore\ P = \frac{H^2\gamma}{2}\frac{\cos^2(\epsilon - \alpha)}{\cos^2\alpha \cos^2\epsilon}A. \quad . \quad . \quad . \quad (f)$$

Now the thrust P combined with the weight of the prism ABF must produce the resultant pressure upon AB.

Then from Fig. 8,

$$V = \frac{H^2\gamma}{2} \tan \alpha \, (1 + \tan \alpha \tan \epsilon)$$

$$= \frac{H^2\gamma}{2} \frac{\sin \alpha \cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon}, \quad . \quad (g)$$

$$E = \sqrt{(V+P \sin \epsilon)^2 + (P \cos \epsilon)^2} = \sqrt{V^2 + P^2 + 2\,VP \sin \epsilon}.$$

Substituting (f) and (g) in this it becomes

$$E = \frac{H^2\gamma}{2} \frac{\cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times$$

$$\sqrt{\sin^2 \alpha + 2 \sin \alpha \sin \epsilon \cos (\epsilon - \alpha) \frac{A}{\cos \epsilon} + \cos^2 (\epsilon - \alpha) \frac{A^2}{\cos^2 \epsilon}},$$

which becomes, by replacing A by its value from (d),

$$E = \frac{H^2\gamma}{2} \frac{\cos (\epsilon - \alpha)}{\cos^2 \alpha \cos \epsilon} \times$$

$$\sqrt{\begin{array}{l} + \sin^2 \alpha \\ + 2 \sin \alpha \sin \epsilon \cos (\epsilon - \alpha) \dfrac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \\ + \cos^2 (\epsilon - \alpha) \left\{ \dfrac{\cos \epsilon - \sqrt{\cos^2 \epsilon - \cos^2 \phi}}{\cos \epsilon + \sqrt{\cos^2 \epsilon - \cos^2 \phi}} \right\}^2 \end{array}}, \quad . \quad (1)$$

which is the general equation for the thrust of earth upon *any plane* within the mass.

To determine the direction of the thrust of the earth, let δ be the angle which the direction of the thrust makes with the horizontal; then, from Fig. 8,

$$\tan \delta = \frac{V}{P \cos \epsilon} + \tan \epsilon.$$

Substituting the values of V and P given above, this becomes

$$\tan\delta = \frac{\sin\alpha\cos\epsilon + \sin\epsilon\cos(\epsilon-\alpha)A}{\cos\epsilon\cos(\epsilon-\alpha)A}, \quad . \quad (1a)$$

where

$$A = \cos\epsilon\,\frac{\cos\epsilon - \sqrt{\cos^2\epsilon - \cos^2\phi}}{\cos\epsilon + \sqrt{\cos^2\epsilon - \cos^2\phi}}. \quad . \quad . \quad (d)$$

Equations (1) and (1a) are readily reduced to more simple forms for special cases. These forms will be found in Part I.

The Plane of Rupture.—Although it is not necessary to know the position of the plane of rupture in order to determine the thrust of the earth, yet it may be of interest to know its position, which can be easily determined as follows:

The plane of rupture will be back of the wall and pass through the heel of the wall. The resultant earth-pressure will make the angle ϕ with the normal to this plane. Now the tangent of the angle which the direction of the resultant earth-pressure on any plane makes with the horizontal is determined from the formula

$$\tan\delta = \frac{\sin\alpha}{\cos(\epsilon-\alpha)A} + \tan\epsilon.$$

If ω represents the angle which the plane of rupture makes with the vertical passing through the heel of the wall, $\alpha = \omega$ and $\delta = \phi + \omega$.

$$\tan(\phi+\omega) = \frac{\sin\omega}{\cos(\epsilon-\omega)A} + \tan\epsilon,$$

from which the value of ω can be determined for any case.

For the case where $\epsilon = \phi$, ϵ being positive with respect to the wall and *negative with respect to the plane of rupture,* the above equation becomes

$$\tan(\phi + \omega) = \frac{\sin \omega}{\cos(\phi + \omega) \cos \phi} - \tan \phi,$$

which is satisfied when $\omega = 90° - \phi$.

For the case where $\epsilon = 0$,

$$\tan(\phi + \omega) = \frac{\sin \omega}{\cos \omega \tan^2\left(45° - \frac{\phi}{2}\right)},$$

which is satisfied when $\omega = 45° - \frac{\phi}{2}$.

Reliability of the Preceding Theory.—The preceding theory is based upon the assumptions that the earth is a homogeneous mass and without cohesion, and the formulas are deduced under the assumption that the surface of the earth is a plane.

All writers on the subject have considered the earth as a homogeneous mass and, with a few exceptions, without cohesion.

Old and recent experiments indicate that cohesion has very little effect upon the pressure of the earth, which explains why it has not been considered by most writers.

The assumption of a plane earth-surface is necessary whenever practical formulas and direct graphical constructions for obtaining the thrust of the earth are obtained. General formulas can be deduced for any character of surface, but they are too complex for practical use. Those graphical constructions which do not require a plane earth-

surface are not direct in their solution of the problem, but require a series of trials to obtain the maximum thrust.

If the earth-surface is not a plane, one can be assumed which will give the thrust of the earth sufficiently exact for all practical purposes.

For unconfined earth no exceptions can be taken to the preceding theory, the assumptions upon which it is based being accepted, and for confined earth the theory must be true when the direction of the principal stress passing through the heel of the wall lies entirely within the earth.

For all cases in which α and ϵ are positive the theories of *Rankine, Winkler, Weyrauch,* and *Mohr* agree and give identical results with the preceding theory, as they should, being founded upon the same assumptions.

When α is negative *Weyrauch* does not consider his theory reliable, and his equations lead to indeterminate results.

Winkler and *Mohr* consider their theories reliable whenever the direction of the principal stress passing through the heel of the wall lies entirely within the earth.

Rankine's method of considering the case where α is negative is equivalent to assuming that the introduction of a wall does not affect the stresses within the mass.

It may be concluded that the preceding theory is perfectly exact when α and ϵ are positive; and when α or ϵ is negative that the stresses obtained will be the maximum which under any circumstances can exist.

For the case where ϵ is negative the stress obtained (which represents the maximum thrust the wall can have against the earth and have equilibrium) will be considerably larger than the actual stress (when a wall is introduced), depending upon the magnitude of ϵ. For small values of ϵ the results will be practically correct. For large values of ϵ

the following method can be employed in determining the thrust of the earth. The method depends upon the *assumption* that the pressure of the earth is normal to the back of the wall. This may or may not be the case, but it appears to be the most consistent assumption to make for this rare and not important case.

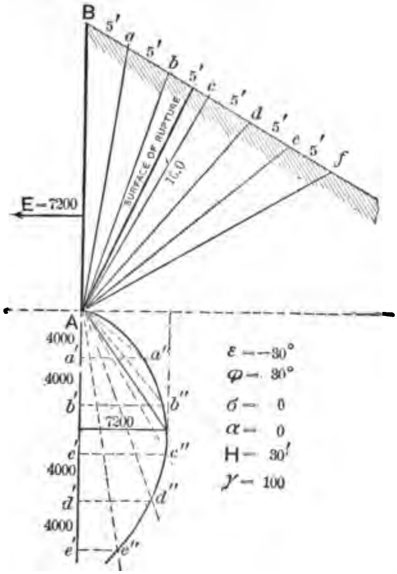

FIG. 8a.

* In Fig. 8*a*, let *AB* be the back of the wall and *Bf* the surface of the earth. Make $Ba = ab = bc = cd =$ etc. Some prism *BAa* or *BAb* or *BAc*, etc., will produce the maximum thrust on the wall; and when this maximum thrust is produced, the resultant pressure on the plane *Aa*

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* See Van Nostrand's Magazine, XVII, 1877, p. 5. "New Constructions in Graphical Statics," by H. T. Eddy, C.E., Ph.D.

or Ab or Ac, etc., will make the angle ϕ with the normal to the plane.

On the vertical line Ad' lay off $Aa'=a'b'=b'c'$, etc., and draw Aa'' making the angle ϕ with the normal to Aa, Ab'' making the angle ϕ with the normal to Ab, etc.; then draw $a'a''$, $b'b''$, etc., perpendicular to AB, and draw a curve through Aa'', b'', c'', etc. Then there will be a maximum distance parallel to $a'a''$ between Ad' and this curve which will be proportional to the thrust of the earth against AB. This maximum distance multiplied by the altitude $Ac \div 2$ and the product by γ, the weight of a cubic foot of earth, will be the pressure of the earth.

This method is perfectly general and can be applied in any case.

If the earth-pressure is assumed to have the direction given by the formulas of the preceding theory, the construction will give the same value of E, the pressure of the earth.

Some writers assume that the direction of E makes the angle $\phi''=\phi$ with the normal to the back of the wall in all cases. This assumption cannot be correct until the wall commences to tip forward, and then it is doubtful that such is the case unless the earth and wall are perfectly dry.

To be on the side of safety in every case, it is better to take the direction of E as given by the above theory.

The construction of Fig. 8a will give the maximum thrust for any assumed direction for any case.

TRAPEZOIDAL WALLS.

It will be assumed that the direction and magnitude of the earth-pressure is known, that the position and extent of the back of the wall and the width of the top are given,

to determine the width of the base for stability against overturning, sliding, and crushing of the material.

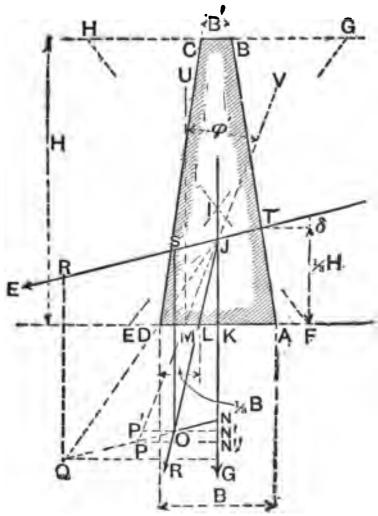

FIG. 9.

Stability against Overturning.—Let $ABCD$, Fig. 9, represent a section of a trapezoidal wall, TR the direction of the earth-thrust, JG the vertical passing through the centre of gravity of the wall, and JO the direction of the resultant pressure on the base AD caused by E and G.

As long as R cuts the base AD, the wall will be stable against overturning. When R takes the direction JQ, the wall may be said to be on the point of overturning; then the factor of safety against overturning is $\frac{QN}{ON}$, where ON is the actual value of E, and QN the value of E required to make the resultant R pass through D.

Stability against Sliding.—Since the wall will not slide

along the surface DA until the resultant R makes an angle with the normal to DA greater than the angle of friction ϕ', the factor of safety against sliding can be obtained as follows: Draw JP making the angle $JMU = \phi'$; then the factor of safety against sliding is $\frac{PN}{ON}$, where PN is the force required in the direction of E to make R make the angle ϕ' with the normal to AD, and ON the actual value of E.

Stability against the Crushing of the Material.—In ordinary practice walls for retaining earth are not of sufficient height to cause very large pressures at their bases, but it is necessary to consider the subject on account of the tendency of the bed-joints to open under certain conditions.

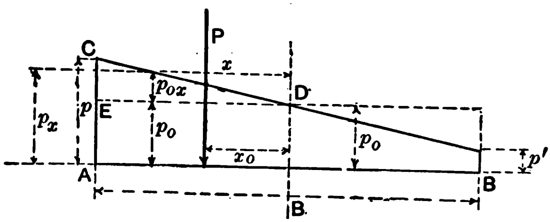

FIG. 10.

Let AB, Fig. 10, represent any bed-joint in the wall, P the vertical resultant pressure upon the joint, and x_0 the distance of the point of application from the centre of the joint.

The intensity of P can be considered as composed of a uniform intensity $p_0 = \frac{P}{B}$, and a uniformly varying intensity p_0', so that $p_x = p_0 + p_0'$. Let a equal the tangent of the angle CDE, then $p_0' = ax$ and $p_x = p_0 + ax$.

The pressure upon a surface (dx)—the joint being considered unity in the dimension normal to the page—is

$$p_x dx = p_0 dx + axdx,$$

and the moment of this about DB is

$$(p_0 dx + axdx)x.$$

The algebraic sum of these moments for values of x between the limits $\pm \frac{B}{2}$ must equal Px_0, or

$$Px_0 = \int_{-\frac{1}{2}B}^{+\frac{1}{2}B} (p_0 xdx + ax^2 dx).$$

Integrating,

$$a = \frac{12x_0 P}{B^3} = \frac{12x_0 p_0}{B^2},$$

and

$$p_x = \frac{B^2 + 12xx_0}{B^2} p_0,$$

or

$$p = \left\{ 1 + \frac{6x_0}{B} \right\} \frac{P}{B};$$

and if x_0 be replaced by $\frac{1}{2}B - Q$, where Q is the distance from A to the point where P cuts the base, (Fig. 11,)

$$p = 2\left(B - \frac{3Q}{B}\right)\frac{P}{B},$$

and

$$p' = 2\left(1 - B + \frac{3Q}{B}\right)\frac{P}{B}.$$

If $Q = \frac{1}{3}B$,

$$p' = 0 \quad \text{and} \quad p = 2p_0,$$

from which it is seen that when R cuts the base outside the middle third, the joint will have a tendency to open at points which are at a maximum distance from R where it cuts the base.

Therefore in no case should the resultant pressure be permitted to cut the base outside the middle third. This makes it unnecessary to consider the stability against overturning.

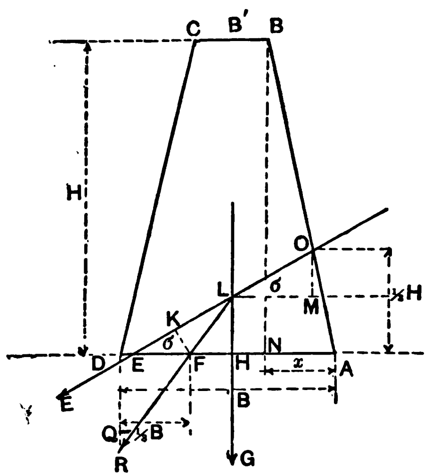

FIG. 11.

Then in designing a wall the following conditions must exist for stability:

I. *The resultant R must cut the base for stability against overturning.*

II. *The resultant R must not make an angle with the normal to the base of the wall greater than the angle of friction ϕ'.*

III. *The resultant R must not cut the base outside of the middle third, in order that there may be no tendency for the bed-joints to open.*

The above three conditions apply to any bed-joint of the wall; but if they are satisfied at the base and the wall has the section shown in Fig. 11, it will not be necessary to consider any joints above the base unless the character of the stone or the bonding is different.

Determination of the width of the base of a retaining-wall under the condition that R cuts the base at a point $\frac{1}{3}B$ from the toe of the wall.

Let H, B', x, δ, and E be given to determine B.

From Fig. 11,

$$KF = \frac{x}{3}\sin\delta + \frac{H}{3}\cos\delta - \frac{2B}{3}\sin\delta,$$

$$HD = \frac{2B^2 + 2BB' - Bx - 2B'x - B'^2}{3(B + B')},$$

$$HF = HD - \frac{B}{3} = \frac{B^2 + BB' - Bx - 2B'x - B'^2}{3(B + B')}.$$

For equilibrium

$$E(KF) = G(HF) = \frac{B + B'}{2}HW(HF).$$

Substituting the values of KF and HF in the above and reducing, it becomes

$$B^2 + B\left(\frac{4E}{HW}\sin\delta + B' - x\right)$$
$$= \frac{2E}{HW}(H\cos\delta + x\sin\delta) + 2B'x + B'^2, \quad . \quad (8)$$

which is the general equation for the width of the base of a trapezoidal wall.

For a rectangular wall $B' = B$.

For a triangular wall $B' = 0$.

For a wall with a vertical front $B' + x = B$ or $B' = B - x$.

For a wall with a vertical back $x = 0$.

Equation (8) is easily transformed to satisfy the requirements of special cases.

The width of the base can be found graphically by assuming a value for B and finding the value of Q; if it is less than $\frac{1}{3}B$ another value of B must be assumed, and so on until Q is equal to or greater than $\frac{1}{3}B$.

Depth of Foundations.—Given the angle of repose ϕ of any earth, to determine the depth to which it is necessary to sink a foundation to support a given load. The surface of the earth is assumed to be horizontal.

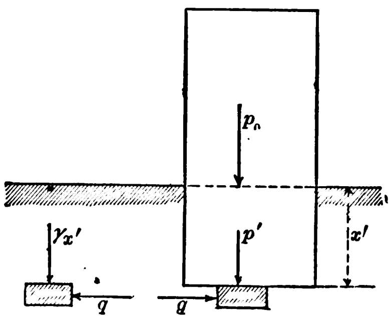

FIG. 12.

CASE I. *When the intensity of the pressure on the base of the foundation is uniform.*

In Fig. 12, let p_0 represent the intensity of the pressure on the base of the foundation.

Now when the masonry is about to sink (see Eq. (c)),

$$\frac{p_0}{q} = \frac{1 + \sin \phi}{1 - \sin \phi} \quad \text{or} \quad q = p_0 \frac{1 - \sin \phi}{1 + \sin \phi}.$$

If x' represents the depth to which the foundation extends below the surface of the earth and γ the weight of a cubic foot of earth, then $\gamma x'$ equals the vertical intensity of the earth-pressure on a plane at the depth of the lowest point of the foundation.

When the wall is on the point of sinking, the earth must be on the point of rising, or

$$\frac{q}{\gamma x'} = \frac{1 + \sin \phi}{1 - \sin \phi},$$

or

$$p_0 = \gamma x' \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 . \quad . \quad . \quad . \quad . \quad (15)$$

In any case p_0 must not have a greater value than that obtained from (15)—

$$x' = \frac{p_0}{\gamma} \left\{ \frac{1 - \sin \phi}{1 + \sin \phi} \right\}^2 = \frac{p_0}{\gamma} \tan^4 \left(45^\circ - \frac{\phi}{2}\right). \quad (16)$$

The value of x' as obtained from (16) is the least allowable value consistent with equilibrium. Since x' is a function of $\tan^4 \left(45^\circ - \frac{\phi}{2}\right)$, care must be taken that ϕ is assumed at its least value. As ϕ becomes smaller the value of x' increases rapidly.

Case II. *When the intensity of the pressure on the base is uniformly varying.*

Let p represent the maximum intensity of the pressure on the earth and p' the minimum intensity; then for

equilibrium p must not exceed the value obtained from the following equation:

$$p = x'\gamma \left\{ \frac{1 + \sin \phi}{1 - \sin \phi} \right\}^2 . \quad . \quad . \quad . \quad . \quad (17)$$

Also, p' must never be *less* than $x'\gamma$; then

$$p_0 = \frac{p+p'}{2} = \frac{x'\gamma}{2} \left\{ 1 + \left(\frac{1+\sin \phi}{1-\sin \phi}\right)^2 \right\} = x'\gamma \frac{1 + \sin^2 \phi}{(1-\sin\phi)^2}, \quad (18)$$

which expresses the maximum value which p_0 can have for the equilibrium of the earth. Solving (18) for x',

$$x' = \frac{p_0}{\gamma} \frac{(1 - \sin \phi)^2}{1 + \sin^2 \phi}, \quad . \quad . \quad . \quad . \quad (19)$$

which is the minimum value x' can have for the equilibrium of the earth.

In order that p' may never be less than $x'\gamma$ the resultant pressure on the base of the foundation must cut the base within a certain distance of the centre of the base. If x_0 equal this distance, then (see page 51)

$$p' = \left(1 - \frac{6x_0}{B}\right) p_0 = x'\gamma.$$

Substituting the value of p_0 from (18) and solving for x_0,

$$x_0 = \frac{1}{3} \frac{\sin \phi}{1 + \sin^2 \phi}, \quad . \quad . \quad . \quad . \quad . \quad (20)$$

which is the *maximum* value x_0 can have, consistent with the stability of the earth.

Abutting Power of Earth.—Let the surface of the earth be horizontal and the body pushing the earth have a verti-

cal face; then at the depth x' the maximum horizontal pressure per unit of area is (see Case I above)

$$q = x'\gamma \frac{1 + \sin \phi}{1 - \sin \phi},$$

and since q varies directly as x', the total thrust P which the earth is capable of resisting is

$$P = \frac{(x')^2\gamma}{2} \frac{1 + \sin \phi}{1 - \sin \phi}. \quad . \quad . \quad . \quad . \quad (21)$$

APPENDIX.

WEYRAUCH'S THEORY OF THE RETAINING-WALL.*

IN the following the earth is supposed without cohesion, and its pressure is determined independently of any arbitrary assumptions as to direction of the earth-pressure, and with sole reference to the three necessary conditions of equilibrium. The single and only supposition, then, is as follows: *That the forces upon any imaginary plane-section through the mass of earth have the same direction.*

This assumption lies at the foundation of *all* theories of earth-pressure against retaining-walls. For those cases, therefore, to which the following discussion does not apply no complete or satisfactory theory is yet possible. In what follows, the ordinary assumption as to the direction of the earth-pressure will be proved to be *incorrect*, except for special cases.

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* *Zeitschrift für Baukunde,* Band I. Heft 2, 1878.

I.

GENERAL RELATIONS.

Let the surface of the earth have any form, and the wall AB, Fig. 1, have any inclination. The earth-pressure makes any angle, δ, with the normal to the wall.

Suppose through the point A the plane AC. Then the weight G of the prism ABC is held in equilibrium by the

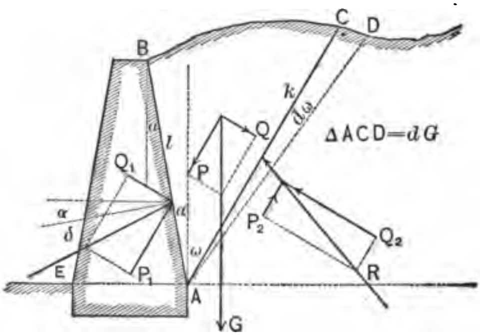


FIG. 1.

reaction of the wall, E, and by the resultant, R, of all the forces acting upon AC.

Now decompose E, G, and R into components parallel and normal to AC; then for every unit in length of the wall, denoting by e, g, and r the lever-arms of E, G, and R respectively with reference to A, the sum of the forces parallel to $AC = 0$, or

$$P - P_1 - P_2 = 0; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the sum of the forces perpendicular to $AC = 0$, or

$$Q + Q_1 - Q_2 = 0\,; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the sum of moments about $A = 0$, or

$$Gg + Ee - Rr = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) was first introduced by Prof. Weyrauch.

Further, according to the theory of friction, if φ is the coefficient of friction for earth on earth,

$$\frac{P_2}{Q_2} \leqq \tan\varphi \text{ or } \frac{P - P_1}{Q + Q_1} \leqq \tan\varphi. \quad . \quad . \quad . \quad (4)$$

If now there is any plane for which

$$P - P_1 = (Q + Q_1)\tan\varphi, \quad . \quad . \quad . \quad (5)$$

this plane AC will be a plane of equilibrium, and $\dfrac{P - P_1}{Q + Q_1}$ will be a maximum, or

$$\frac{d\left(\dfrac{P - P_1}{Q + Q_1}\right)}{d\omega} = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This plane is designated as the "surface of rupture."

From Fig. 1, for every position of AC,

$$P = G\cos\omega, \qquad Q = G\sin\omega,$$
$$P_1 = E\sin(\omega + \alpha + \delta), \qquad Q_1 = E\cos(\omega + \alpha + \delta).$$

Substituting the above values of P, P_1, Q, and Q_1 in equation (5), it becomes

$$G\cos\omega - E\sin(\omega + \alpha + \delta)$$
$$= [G\sin\omega + E\cos(\omega + \alpha + \delta)]\tan\varphi\,;$$

and when ω refers to the surface of rupture, the earth-pressure upon AB becomes

$$E = \frac{\cos\omega - \sin\omega\tan\varphi}{\sin(\omega+\alpha+\delta) + \cos(\omega+\alpha+\delta)\tan\varphi} G.$$

Substituting the value of $\tan\varphi$ or $\frac{\sin\varphi}{\cos\varphi}$, this becomes

$$E = \frac{\cos\varphi\cos\omega - \sin\omega\sin\varphi}{\sin(\omega+\alpha+\delta)\cos\varphi + \cos(\omega+\alpha+\delta)\sin\varphi} G,$$

which becomes

$$E = \frac{\cos(\varphi+\omega)}{\sin(\varphi+\omega+\alpha+\delta)} G. \quad . \quad . \quad (7)$$

In order to refer to the surface of rupture, the following relation must exist:

$$\frac{d\left(\dfrac{G\cos\omega - E\sin(\omega+\alpha+\delta)}{G\sin\omega + E\cos(\omega+\alpha+\delta)}\right)}{d\omega} = 0. \quad (7a)$$

Performing the differentiation indicated in the equation (7a), considering G and ω as the variables, it becomes

$$\frac{\begin{array}{l}+[dG\cos\omega - \sin\omega d\omega\, G - E\cos(\omega+\alpha+\delta)d\omega]\,[G\sin\omega + E\cos(\omega+\alpha+\delta)] \\ -[dG\sin\omega + \cos\omega d\omega\, G - E\sin(\omega+\alpha+\delta)d\omega]\,[G\cos\omega - E\sin(\omega+\alpha+\delta)]\end{array}}{[G\sin\omega + E\cos(\omega+\alpha+\delta)]^2 d\omega} =$$

$$= 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7b).$$

dividing by $d\omega$, this becomes

$$\frac{\begin{array}{l}+\left[\dfrac{dG\cos\omega}{d\omega} - [G\sin\omega + E\cos(\omega+\alpha+\delta)]\right][G\sin\omega + E\cos(\omega+\alpha+\delta)] \\ -\left[\dfrac{dG\sin\omega}{d\omega} + [G\cos\omega - E\sin(\omega+\alpha+\delta)]\right][G\cos\omega - E\sin(\omega+\alpha+\delta)]\end{array}}{[G\sin\omega + E\cos(\omega+\alpha+\delta)]^2} =$$

$$= 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7c)$$

or

$$\frac{\begin{array}{l}+\dfrac{dG\cos\omega}{d\omega}[G\sin\omega+E\cos(\omega+\alpha+\delta)]-[G\sin\omega+E\cos(\omega+\alpha+\delta)]^2\\-\dfrac{dG\sin\omega}{d\omega}[G\cos\omega-E\sin(\omega+\alpha+\delta)]-[G\cos\omega-E\sin(\omega+\alpha+\delta)]^2\end{array}}{[G\sin\omega+E\cos(\omega+\alpha+\delta)]^2}=$$

$$=0. \quad (7d$$

Now, since

$$\cos\omega\cos(\omega+\alpha+\delta)+\sin\omega\sin(\omega+\alpha+\delta)=\cos(\alpha+\delta)$$

and $\sin^2\omega+\cos^2\omega=1,$

by clearing of fractions this becomes

$$-\frac{EdG\cos(\alpha+\delta)}{d\omega}+G^2-2GE\sin(\alpha+\delta)+E^2=0. \quad (7e)$$

Now since $dG=\frac{1}{2}k\,.\,d\omega\,.\,k\gamma$, equation (7e) reduces to

$$G^2-2GE\sin(\alpha+\delta)-\frac{Ek^2\gamma\cos(\alpha+\delta)}{2}+E^2=0, \quad (7f)$$

which becomes, after dividing by GE,

$$\frac{G}{E}-2\sin(\alpha+\delta)-\frac{k^2\gamma\cos(\alpha+\delta)}{2G}+\frac{E}{G}=0. \quad (8)$$

Substituting the value of $\frac{E}{G}$ from equation (7), transposing and multiplying by two, equation (8) reduces to

$$\frac{2\sin(\phi+\alpha+\omega+\delta)}{\cos(\phi+\omega)}-4\sin(\alpha+\delta)+\frac{2\cos(\phi+\omega)}{\sin(\phi+\omega+\alpha+\delta)}=\frac{k^2\gamma\cos(\alpha+\delta)}{G}, \quad (8a)$$

whence

$$G = \frac{k^2\gamma \cos(a+\delta)}{\dfrac{2\sin(\phi+\omega+a+\delta)}{\cos(\phi+\omega)} - 4\sin(a+\delta) + \dfrac{2\cos(\phi+\omega)}{\sin(\phi+\omega+a+\delta)}}, \quad \ldots \ldots \quad (8b)$$

which reduces to

$$G = \frac{\cos(\phi+\omega)\sin(\phi+\omega+a+\delta)\cos(a+\delta)k^2\gamma}{2[\sin^2(\phi+\omega+a+\delta) - 2\sin(a+\delta)\cos(\phi+\omega)\sin(\phi+\omega+a+\delta) + \cos^2(\phi+\omega)]}. \quad (8c)$$

Since

$$\sin(\varphi+\omega+\alpha+\delta) = \sin(\varphi+\omega)\cos(\alpha+\delta) + \cos(\varphi+\omega)\sin(\alpha+\delta),$$

the parenthetical portion of the denominator becomes

$$\begin{aligned} &\sin^2(\varphi+\omega)\cos^2(\alpha+\delta) \\ &\quad + 2\sin(\alpha+\delta)\cos(\varphi+\omega)\sin(\varphi+\omega)\cos(\alpha+\delta) \\ &\quad + \cos^2(\varphi+\omega)\sin^2(\alpha+\delta) \\ &\quad - 2\sin(\alpha+\delta)\cos(\varphi+\omega)\sin(\varphi+\omega)\cos(\alpha+\delta) \\ &\quad - 2\sin(\alpha+\delta)\cos(\varphi+\omega)\cos(\varphi+\omega)\sin(\alpha+\delta) \\ &\quad + \cos^2(\varphi+\omega), \end{aligned}$$

or

$$\begin{aligned} &\sin^2(\varphi+\omega)\cos^2(\alpha+\delta) \\ &\qquad - 2\sin^2(\alpha+\delta)\cos^2(\varphi+\omega) \\ &\qquad + \sin^2(\alpha+\delta)\cos^2(\varphi+\omega) + \cos^2(\varphi+\omega), \end{aligned}$$

or $$\sin^2(\varphi+\omega)\cos^2(\alpha+\delta) - \cos^2(\varphi+\omega)\sin^2(\alpha+\delta) + \cos^2(\varphi+\omega),$$

or $$\sin^2(\varphi+\omega)\cos^2(\alpha+\delta) + \cos^2(\varphi+\omega)[1-\sin^2(\alpha+\delta)],$$

or $$\sin^2(\varphi+\omega)\cos^2(\alpha+\delta) + \cos^2(\varphi+\omega)\cos^2(\alpha+\delta),$$

or $$\cos^2(\alpha+\delta)[\sin^2(\varphi+\omega) + \cos^2(\varphi+\omega)],$$

which equals $\cos^2(\alpha+\delta)$, and equation (8c) becomes, after dividing by $\cos(\alpha+\delta)$ and factoring,

$$G=\frac{\cos(\varphi+\omega)\sin(\varphi+\omega+\alpha+\delta)}{\cos(\alpha+\delta)}\cdot\frac{k^2\gamma}{2}=\text{Function }\gamma, \quad (9)$$

from which

$$\sin(\varphi+\omega+\alpha+\delta)=\frac{2G}{k^2\gamma}\cdot\frac{\cos(\alpha+\delta)}{\cos(\varphi+\omega)},$$

which being substituted in equation (7) gives

$$E=\frac{G\cos(\varphi+\omega)}{\dfrac{2G\cos(\alpha+\delta)}{k^2\gamma\cos(\varphi+\omega)}}=\frac{\cos^2(\varphi+\omega)}{\cos(\alpha+\delta)}\cdot\frac{k^2\gamma}{2}. \quad . \quad (10)$$

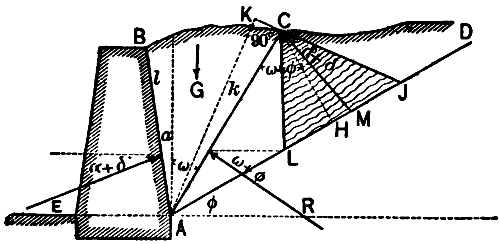


FIG. 2.

And, since the sum of the horizontal components of E, G, and R must be equal to 0, or Fig. 2,

$$E\cos(\alpha+\delta)=R\cos(\omega+\varphi),$$

and
$$R=E\frac{\cos(\alpha+\delta)}{\cos(\omega+\varphi)};$$

which becomes, after substituting the value of E from equation (10),

$$R = \cos(\varphi+\omega)\frac{k^2\gamma}{2}. \quad . \quad . \quad . \quad . \quad (11)$$

Let AD, Fig. 2, be the natural slope of the ground. From C let fall the perpendicular CH, and draw CJ, making the angle $(\alpha+\delta)$ with CH; then

$$CH = k\cos(\varphi+\omega), \qquad AJ = \frac{\sin(\varphi+\omega+\alpha+\delta)}{\cos(\alpha+\delta)}k.$$

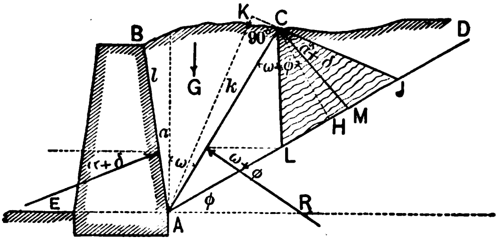


FIG. 2.

The expression for AJ is obtained in the following manner (Fig. 2):

$$CH = k\cos(\varphi+\omega), \qquad AH = k\sin(\varphi+\omega),$$
$$HJ : CH :: \sin(\alpha+\delta) : \cos(\alpha+\delta),$$

and $$HJ = \frac{CH\sin(\alpha+\delta)}{\cos(\alpha+\delta)} = \frac{\cos(\varphi+\omega)\sin(\alpha+\delta)}{\cos(\alpha+\delta)}k,$$

$$AH + HJ = AJ$$
$$= \frac{\sin(\varphi+\omega)\cos(\alpha+\delta)+\cos(\varphi+\omega)\sin(\alpha+\delta)}{\cos(\alpha+\delta)}k,$$

which reduces to

$$AJ = \frac{\sin(\varphi+\omega+\alpha+\delta)}{\cos(\alpha+\delta)}k;$$

and hence, according to equation (9),

$$G = \text{Func.}\,\gamma = \gamma \Delta ACJ. \quad . \quad . \quad . \quad . \quad (12)$$

Also, if AK is perpendicular to CJ,

$$\frac{CH}{AK} = \frac{k\cos(\varphi+\omega)}{k\sin(\varphi+\omega+\alpha+\delta)} = \frac{E}{G};$$

and if JL is made equal to JC, then, since the perpendicular from L upon CJ is equal to CH,

$$\frac{\Delta CJL}{\Delta CJA} = \frac{CH}{AK} = \frac{E}{G},$$

or

$$E = \gamma \Delta CJL. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If, finally, $AM = AC$,

$$\Delta ACM = \frac{AM \,.\, CH}{2} = \tfrac{1}{2}k^2\cos(\varphi+\omega),$$

or

$$R = \gamma \Delta ACM. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

All these geometrical results may be summed up as follows:

Draw from the highest point C of the surface of rupture a line CJ, which makes with the normal CH to the natural slope the angle $\alpha + \delta$, or the angle which the earth-pressure makes with the horizontal; then the ΔACJ is

equal in area to the ΔABC, the prism of rupture. Then lay off $JL = JC$ and $AM = AC$ and draw CL and CM; then for every unit in length of the wall the following relations exist:

$$\left.\begin{array}{ll}\text{Weight of prism of rupture,} & G = \gamma\Delta CAJ;\\ \text{Earth-pressure upon wall,} & E = \gamma\Delta CJL;\\ \text{Reaction of the surface of rupture,} & R = \gamma\Delta CAM.\end{array}\right\} \quad (14a)$$

The first two relations were first made known by Rebhahn in 1871, for $\delta = 0$ or φ.

$$\text{Since, now,}\quad G : E : R = AJ : JC : CA, \quad . \quad . \quad . \quad (15)$$

it can be asserted that—

The weight of the prism of rupture and the reactions of the wall and of the surface of rupture are to each other as the three sides of the ΔACJ.

Thus far no assumption whatever has been made as to the value of the angle δ. This is determined by equation (3), which, in all theories following Coulomb's method, does not occur.

II.

PLANE EARTH-SURFACE INCLINED.

ADOPT in this case the notation of Fig. 3, and let E be first determined for any value of δ.

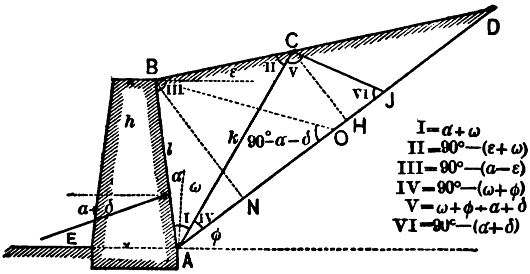


FIG. 3.

If AC is the surface of rupture, then $\Delta ABC = \Delta ACJ$; or, since

$$\frac{AB}{AC} = \frac{\sin\ \mathrm{II}}{\sin\ \mathrm{III}}, \qquad AB = AC\frac{\sin\ \mathrm{II}}{\sin\ \mathrm{III}}.$$

In like manner, $AJ = AC\dfrac{\sin\ V}{\sin\ VI}$.

But since $\Delta ABC = \Delta ACJ$,

$$AB\ .\ AC \sin I = AJ\ .\ AC \sin IV; \quad . \quad . \qquad (16)$$

or

$$\frac{\sin I \sin II}{\sin III} = \frac{\sin IV \sin V}{\sin VI}; \quad . \quad . \quad . \qquad (16a)$$

or, finally,

$$\sin(\alpha+\omega)\cos(\varepsilon+\omega)\cos(\alpha+\delta)$$
$$=\sin(\varphi+\omega+\alpha+\delta)\cos(\varphi+\omega)\cos(\alpha-\varepsilon). \quad (16b)$$

Further, from Fig. 3, if BN is perpendicular to AD,

$$\Delta ADB = 2\Delta AJC + \Delta JDC,$$

or

$$AD \,.\, BN = 2AJ \,.\, CH + JD \,.\, CH;$$

and since

$$\frac{BN}{CH} = \frac{BO}{CJ} = \frac{OD}{JD},$$

and

$$AD \,.\, OD = JD\,(AJ + AD),$$
$$AD\,(AD - AO) = (AD - AJ)\,(AJ + AD),$$

whence

$$AO \,.\, AJ = AJ \,.\, AD. \quad . \quad . \quad . \quad (17)$$

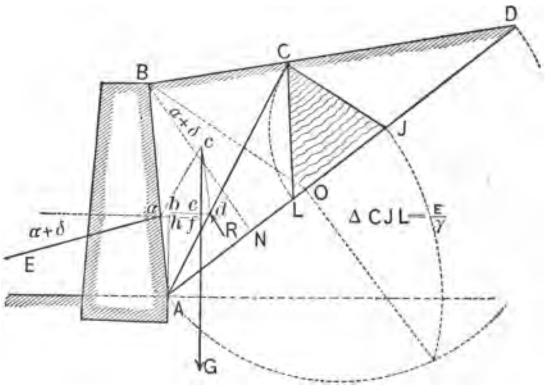


FIG. 3′.

Upon this relation rests the well-known construction of Poncelet for the earth-pressure. Draw (Fig. 3′) BN perpendicular to the natural slope AD; draw BO, making the same angle with BN that E makes with the horizontal, and

then determine the point J so that equation (17) is fulfilled, that is, make AJ a mean proportional between AO and AD; then draw JC parallel to OB. Thus the surface of rupture AC is found, and use can now be made of the relations already deduced in I.

In order to determine J (A, O, and D being given), there are several methods, one of which is indicated in the figure. In all these constructions δ *is assumed.*

Now from equation (13), $E = \frac{1}{2}\gamma\,\overline{JC}^2 \cos(\alpha + \delta)$,

but

$$\frac{CJ}{BO} = \frac{AD - AJ}{AD - AO} = \frac{AD - \sqrt{AD \cdot AO}}{AD - AO} = \frac{1 - \sqrt{\frac{AO}{AD}}}{1 - \frac{AO}{AD}}.$$

Let $n = \sqrt{\frac{AO}{AD}}$, then $CJ = \frac{1-n}{1-n^2}BO = \frac{BO}{1+n}$.

From Fig. 3,

$$\frac{AO}{AB} = \frac{\sin(\varphi + \delta)}{\cos(\alpha + \delta)}, \qquad \frac{AB}{AD} = \frac{\sin(\varphi - \varepsilon)}{\cos(\alpha - \varepsilon)};$$

and the multiplication of these equations gives

$$n = \sqrt{\frac{\sin(\varphi + \delta)\sin(\varphi - \varepsilon)}{\cos(\alpha + \delta)\cos(\alpha - \varepsilon)}}. \qquad . \quad . \quad (18)$$

If $AB = l$, $\qquad BO = \frac{\cos(\varphi - \alpha)}{\cos(\alpha + \delta)}l$;

and by substitution of BO and n in the value for CJ, and of CJ in that for E,

$$E = \left[\frac{\cos(\phi - \alpha)}{n+1}\right]^2 \frac{l^2\gamma}{2\cos(\alpha+\delta)} = \left[\frac{\cos(\phi - \alpha)}{(n+1)\cos\alpha}\right]^2 \frac{h^2\gamma}{2\cos(\alpha+\delta)}. \quad . \quad . \quad (19)$$

For the special case of the earth-surface parallel to the angle of repose, $\varepsilon = \varphi$, $n = 0$, and

$$E = \frac{\cos^2(\varphi - \alpha)}{\cos(\alpha+\delta)} \frac{l^2\gamma}{2} = \left[\frac{\cos(\varphi-\alpha)}{\cos\alpha}\right]^2 \frac{h^2\gamma}{2\cos(\alpha+\delta)}. (20)$$

These formulæ hold good for any value of δ. But the angle δ is determined by equation (3). In order to insert e and r in this formula, the points of application of E and R must be known. The angles δ and ω are connected by the relations in (16*b*), in which there are no other unknown quantities. Since now δ, according to the single assumption of Prof. Weyrauch's theory, is independent of the height, so also is ω, and then for variable h equations (19) and (11) become

$$E = Cl^2, \qquad R = C_1k^2,$$
$$dE = 2Cldl, \qquad dR = 2C_1kdk.$$

Let x and z equal the distance of the point of application of E and R from A, respectively. Now considering the top as the origin or centre of moments,

$$E(l - x) = 2C\int_0^l l^2dl, \qquad R(k - z) = 2C_1\int_0^k k^2dk,$$

and therefore $x = \frac{1}{3}l$ and $z = \frac{1}{3}k$.

Now G must act through the centre of gravity of the ΔABC, and it has been already proved that the points

of application of E and R are at distances $\frac{1}{3}l$ and $\frac{1}{3}k$ respectively above A; hence (Fig. 3′) $ah = ed$ and $hf = g = bd - ah = \frac{1}{3}k \sin \omega - \frac{1}{3}l \sin \alpha$.

Substituting these values in equation (3) and referring to equation (15),

$$AB\,(CJ \cos \delta - AJ \sin \alpha) = AC\,(AC \cos \phi - AJ \sin \omega), \quad . \quad . \quad . \quad (22)$$

or

$$\sin II\,(\sin IV \cos \delta - \sin V \sin \alpha) = \sin III\,(\sin VI \cos \phi - \sin V \sin \omega), \quad (22a)$$

or

$$\cos(\epsilon + \omega)\,[\cos(\phi + \omega)\cos \delta - \sin(\phi + \omega + \alpha + \delta)\sin \alpha]$$
$$= \cos(\alpha - \epsilon)\,[\cos(\alpha + \delta)\cos \phi - \sin(\phi + \omega + \alpha + \delta)\sin \omega]. \quad . \quad . \quad (22b)$$

By means of the two equations (16*b*) and (22*b*) the two unknown quantities δ and ω are completely determined. As soon as these are known, E can be found from equation (19) or (20). Also by the relations in equations (16) and (22), or (16*a*) and (22*b*), the surface of rupture and direction of the earth-pressure may be determined, and can therefore be found by a graphical construction.

III.

HORIZONTAL EARTH-SURFACE.

FOR this most important practical case it is simply necessary to make $\varepsilon = 0$ in equation (19). The proper values of δ and ω in this case are found from (16*b*) and (22*b*).

Making $\varepsilon = 0$ in equation (22*b*), it becomes

$$\cos\omega\,[\cos(\varphi+\omega)\cos\delta - \sin(\varphi+\omega+\alpha+\delta)\sin\alpha]$$
$$-\cos\alpha\,[\cos(\alpha+\delta)\cos\varphi - \sin(\varphi+\omega+\alpha+\delta)\sin\omega]=0.$$

Since

$$\begin{aligned}\sin(\varphi+\omega+\alpha+\delta) &= \sin(\varphi+\omega)\cos(\alpha+\delta)\\ &\quad + \cos(\varphi+\omega)\sin(\alpha+\delta),\\ \cos(\alpha+\delta) &= \cos\alpha\cos\delta - \sin\alpha\sin\delta,\\ \text{and}\qquad \sin(\alpha+\delta) &= \sin\alpha\cos\delta + \cos\alpha\sin\delta,\end{aligned}$$

the above expression becomes

$$\left.\begin{aligned}&\cos\omega\cos\delta\cos(\varphi+\omega)\\ &-\cos\omega\sin\alpha\cos\alpha\cos\delta\sin(\varphi+\omega)\\ &\qquad\qquad+\cos\omega\sin^2\alpha\sin\delta\sin(\varphi+\omega)\\ &-\cos\omega\sin\alpha\cos\alpha\sin\delta\cos(\varphi+\omega)\\ &\qquad\qquad-\cos\omega\sin^2\alpha\cos\delta\cos(\varphi+\omega)\\ &-\cos\alpha\cos\varphi\cos(\alpha+\delta)\\ &+\cos^2\alpha\sin\omega\cos\delta\sin(\varphi+\omega)\\ &\qquad\qquad-\cos\alpha\sin\omega\sin\alpha\sin\delta\sin(\varphi+\omega)\\ &+\cos^2\alpha\sin\omega\sin\delta\cos(\varphi+\omega)\\ &\qquad\qquad+\cos\alpha\sin\omega\sin\alpha\cos\delta\cos(\varphi+\omega)\end{aligned}\right\}=0,$$

which reduces to

$$\left.\begin{array}{l}\cos\omega\cos(\varphi+\omega)\cos\delta \\ -\sin\alpha\cos\alpha\,[\sin(\varphi+\omega)\cos\omega-\cos(\varphi+\omega)\sin\omega]\cos\delta \\ -\sin\alpha\cos\alpha\,[\cos(\varphi+\omega)\cos\omega+\sin(\varphi+\omega)\sin\omega]\sin\delta \\ +[\sin^2\alpha\sin(\varphi+\omega)\cos\omega+\cos^2\alpha\cos(\varphi+\omega)\sin\omega]\sin\delta \\ +[\cos^2\alpha\sin(\varphi+\omega)\sin\omega-\sin^2\alpha\cos(\varphi+\omega)\cos\omega]\cos\delta \\ -\cos^2\alpha\cos\varphi\cos\delta+\sin\alpha\cos\alpha\cos\varphi\sin\delta\end{array}\right\} \\ =0. \qquad (22c)$$

The expression in the first parenthesis is equal to $\sin\varphi$, in the second to $\cos\varphi$. If in the third $\cos^2\alpha = 1 - \sin^2\alpha$, and in the fourth $\sin^2\alpha = 1 - \cos^2\alpha$, equation ($22c$) becomes

$$\left.\begin{array}{r}+\cos\omega\cos(\varphi+\omega)\cos\delta-\sin\alpha\cos\alpha\cos\delta\sin\varphi \\ -\sin\alpha\cos\alpha\sin\delta\cos\varphi \\ +\sin\delta\sin^2\alpha\sin(\varphi+\omega)\cos\omega+\sin\delta\sin\omega\cos(\varphi+\omega) \\ -\sin^2\alpha\sin\omega\sin\delta\cos(\varphi+\omega) \\ +\cos\delta\cos^2\alpha\sin(\varphi+\omega)\sin\omega-\cos\delta\cos\omega\cos(\varphi+\omega) \\ +\cos^2\alpha\cos\delta\cos\omega\cos(\varphi+\omega) \\ -\cos^2\alpha\cos\varphi\cos\delta+\sin\alpha\cos\alpha\cos\varphi\sin\delta\end{array}\right\}=0.$$

Reducing and dividing by $\cos\delta$,

$$\left.\begin{array}{r}-\sin\alpha\cos\alpha\sin\varphi+\sin^2\alpha\cos\omega\sin(\varphi+\omega)\tan\delta \\ +\sin\omega\cos(\varphi+\omega)\tan\delta \\ -\sin^2\alpha\sin\omega\cos(\varphi+\omega)\tan\delta \\ +\cos^2\alpha\sin\omega\sin(\varphi+\omega) \\ +\cos^2\alpha\cos\omega\cos(\varphi+\omega)-\cos^2\alpha\cos\varphi\end{array}\right\}=0.$$

Since

$$\cos\omega\sin(\varphi+\omega)-\sin\omega\cos(\varphi+\omega)=\sin\varphi$$

2

and

$$\sin\omega\sin(\varphi+\omega)+\cos\omega\cos(\varphi+\omega)=\cos\varphi,$$

this reduces to

$$-\sin\alpha\cos\alpha\sin\varphi+\sin^2\alpha\sin\varphi\tan\delta$$
$$+\sin\omega\cos(\varphi+\omega)\tan\delta=0;$$

and since

$$\cos(\varphi+\omega)\sin\omega=\tfrac{1}{2}\sin(2\omega+\varphi)-\tfrac{1}{2}\sin\varphi,$$

this becomes

$$\tan\delta=\frac{2\sin\alpha\cos\alpha\sin\varphi}{2\sin^2\alpha\sin\varphi+\sin(2\omega+\varphi)-\sin\varphi};$$

and since

$$\sin\alpha\cos\alpha=\tfrac{1}{2}\sin 2\alpha \quad\text{and}\quad 1-2\sin^2\alpha=\cos 2\alpha,$$

this reduces to

$$\tan\delta=\frac{\sin\varphi\sin 2\alpha}{\sin(2\omega+\varphi)-\sin\varphi\cos 2\alpha}. \quad . \quad (23)$$

This equation, therefore, expresses the condition that the *"sum of the moments of E, G, and R is zero."*

Substituting $\dfrac{\sin\delta}{\cos\delta}$ for $\tan\delta$ in equation (23), clearing of fractions and factoring,

$$\sin\delta\sin(2\omega+\varphi)-\sin\delta\sin\varphi\cos 2\alpha=\sin\varphi\cos\delta\sin 2\alpha,$$

or

$$\sin\delta \sin(2\omega + \varphi) = \sin\varphi \cos\delta \sin 2\alpha + \sin\varphi \sin\delta \cos 2\alpha.$$

Since $\quad \cos\delta \sin 2\alpha + \sin\delta \cos 2\alpha = \sin(2\alpha + \delta),$

this becomes

$$\sin\delta \sin(2\omega + \varphi) = \sin\varphi \sin(2\alpha + \delta). \quad . \quad (24)$$

In order to determine ω it is only necessary to make $\varepsilon = 0$ in equation (16b) express $\sin(\varphi + \omega + \alpha + \delta)$ in terms of sin and cos $(\varphi + \omega)$ and $(\alpha + \delta)$, and then the sin and cos of $(\alpha + \delta)$ in terms of the sin and cos of α and δ. Making $\varepsilon = 0$ in equation (16b), it becomes

$$\begin{aligned} &\sin(\alpha + \omega)\cos(\alpha + \delta)\cos\omega \\ &\qquad = \sin(\varphi + \omega + \alpha + \delta)[\cos(\varphi + \omega)\cos\alpha]. \quad (24a) \end{aligned}$$

Since

$$\begin{aligned} \sin(\varphi + \omega + \alpha + \delta) &= \sin(\varphi + \omega)\cos(\alpha + \delta) \\ &\qquad + \cos(\varphi + \omega)\sin(\alpha + \delta) \\ \sin(\alpha + \delta) &= \sin\alpha\cos\delta + \cos\alpha\sin\delta \\ \cos(\alpha + \delta) &= \cos\alpha\cos\delta - \sin\alpha\sin\delta; \end{aligned}$$

hence

$$\begin{aligned} \sin(\varphi + \omega + \alpha + \delta) = \sin(\varphi + \omega)\cos\alpha\cos\delta \\ - \sin(\varphi + \omega)\sin\alpha\sin\delta \\ + \cos(\varphi + \omega)\sin\alpha\cos\delta \\ + \cos(\varphi + \omega)\cos\alpha\sin\delta, \end{aligned}$$

and equation (24a) reduces to

$$\left.\begin{array}{l}\cos\omega\sin(\alpha+\omega)\cos\alpha\cos\delta\\ \qquad\qquad -\cos\omega\sin(\alpha+\omega)\sin\alpha\sin\delta\\ -\cos^2\alpha\cos(\varphi+\omega)\sin(\varphi+\omega)\cos\delta\\ \quad+\cos\alpha\cos(\varphi+\omega)\sin(\varphi+\omega)\sin\alpha\sin\delta\\ -\cos\alpha\cos^2(\varphi+\omega)\sin\alpha\cos\delta\\ -\cos^2\alpha\cos^2(\varphi+\omega)\sin\delta\end{array}\right\}=0. \quad (24b)$$

Dividing by $\cos\delta$,

$$\left.\begin{array}{l}\cos\alpha\cos\omega\sin(\alpha+\omega)\\ \qquad\qquad -\cos\omega\sin\alpha\sin(\alpha+\omega)\tan\delta\\ -\cos^2\alpha\cos(\varphi+\omega)\sin(\varphi+\omega)\\ \quad+\cos\alpha\sin\alpha\cos(\varphi+\omega)\sin(\varphi+\omega)\tan\delta\\ -\cos\alpha\sin\alpha\cos^2(\varphi+\omega)\\ -\cos^2\alpha\cos^2(\varphi+\omega)\tan\delta\end{array}\right\}=0. \quad (24c)$$

Since

$\cos\alpha\cos\omega\sin(\alpha+\omega)$ equals, by expanding $\sin(\alpha+\omega)$, $\sin\alpha\cos\alpha\cos^2\omega+\sin\omega\cos\omega\cos^2\alpha$, and likewise

$$-\cos\omega\sin\alpha\sin(\alpha+\omega)\tan\delta=-\cos^2\omega\sin^2\alpha\tan\delta$$
$$-\cos\alpha\sin\alpha\cos\omega\sin\omega\tan\delta,$$

equation (24c) becomes

$$\left.\begin{array}{l}-\sin\alpha\cos\alpha\,[\cos^2(\varphi+\omega)-\cos^2\omega]\\ -\cos^2\alpha\,[\sin(\varphi+\omega)\cos(\varphi+\omega)-\sin\omega\cos\omega]\\ -[\cos^2\alpha\cos^2(\varphi+\omega)+\sin^2\alpha\cos^2\omega]\tan\delta\\ +\sin\alpha\cos\alpha\,[\sin(\varphi+\omega)\cos(\varphi+\omega)\\ \qquad\qquad -\sin\omega\cos\omega]\tan\delta\end{array}\right\}=0. \quad (24d)$$

Now

$$\cos^2(\varphi+\omega)-\cos^2\omega=\frac{\cos 2(\varphi+\omega)-\cos 2\,\omega}{2},$$

which equals

$$\frac{2\sin\frac{1}{2}[2\omega-(2\varphi+2\omega)]\,\sin\frac{1}{2}[2\omega+(2\varphi+2\omega)]}{2}$$

$$=\frac{2\sin(-\varphi)\sin(2\omega+\varphi)}{2},$$

or $$-\sin(2\omega+\varphi)\sin\varphi,$$

and

$$\sin(\varphi+\omega)\cos(\varphi+\omega)-\sin\omega\cos\omega$$
$$=\tfrac{1}{2}\sin 2(\varphi+\omega)-\tfrac{1}{2}\sin 2\omega;$$

also,

$$\sin\alpha\cos\alpha=\frac{\sin 2\alpha}{2},\ \text{and}\ \cos^2\alpha=\frac{\cos 2\alpha}{2}+\tfrac{1}{2}.$$

Hence, after multiplying by 2, equation (24*d*) reduces to

$$\left.\begin{array}{l}
\sin 2\alpha\sin(2\omega+\varphi)\sin\varphi \\
\quad-\cos 2\alpha\,\tfrac{1}{2}\sin 2(\varphi+\omega)+\cos 2\alpha\,\tfrac{1}{2}\sin 2\,\omega \\
-\tfrac{1}{2}\sin 2(\varphi+\omega)+\tfrac{1}{2}\sin 2\omega \\
-\tan\delta\cos 2\alpha\cos^2(\varphi+\omega)-\cos^2(\varphi+\omega)\tan\delta \\
-2\tan\delta\sin^2\alpha\cos^2\omega \\
\quad\quad+\sin 2\alpha\sin(\varphi+\omega)\cos(\varphi+\omega)\tan\delta \\
-\sin 2\alpha\sin\omega\cos\omega\tan\delta
\end{array}\right\}=0. \quad (24e)$$

Now

$$-2\tan\delta\sin^2\alpha\cos^2\omega = [\text{since } \sin^2\alpha = 1-\cos^2\alpha]$$
$$-[\cos^2\omega - \cos^2\alpha\cos^2\omega]\,2\tan\delta,$$

which equals

$$-\underline{2\cos^2\omega\tan\delta} + 2\tan\delta\cos^2\alpha\cos^2\omega.$$

Also,

$$-\frac{\cos 2\alpha\sin 2(\varphi+\omega)}{2} + \frac{\cos 2\alpha\sin 2\omega}{2}$$
$$= -\cos 2\alpha\left[\frac{\sin 2(\varphi+\omega) - \sin 2\omega}{2}\right]$$
$$= -\frac{\cos 2\alpha[2\sin\varphi\cos(2\omega+\varphi)]}{2}$$
$$= -\underline{\cos 2\alpha\cos(2\omega+\varphi)\sin\varphi},$$

and

$$-\frac{\sin 2(\varphi+\omega)}{2} + \frac{\sin 2\omega}{2} = -\frac{\sin 2(\varphi+\omega) - \sin 2\omega}{2}$$
$$= -\frac{2\sin\frac{1}{2}(2\varphi+2\omega-2\omega)\cos\frac{1}{2}(2\varphi+2\omega+2\omega)}{2}$$
$$= -\underline{\sin\varphi\cos(2\omega+\varphi)},$$

and

$$-\tan\delta\cos 2\alpha\cos^2(\varphi+\omega) + 2\tan\delta\cos^2\alpha\cos^2\omega$$
$$= \left(\text{by making } \cos^2\alpha = \frac{\cos 2\alpha}{2} + \tfrac{1}{2}\right)$$
$$-\tan\delta\cos 2\alpha[\cos^2(\varphi+\omega) - \cos^2\omega] + \tan\delta\cos^2\omega,$$

or $\quad \underline{\tan\delta\cos 2\alpha\sin(2\omega+\varphi)\sin\varphi} + \tan\delta\cos^2\omega,$

Also, $$\begin{aligned} &- \cos^2(\varphi + \omega) \tan \delta + \tan \delta \cos^2 \omega \\ &= - \tan \delta\,[\cos^2(\varphi + \omega) - \cos^2 \omega] \\ &= \underline{\sin \varphi \sin (2\omega + \varphi) \tan \delta.} \end{aligned}$$

Also,

$$\begin{aligned} &\tan \delta \sin 2\alpha \sin (\varphi + \omega) \cos (\varphi + \omega) \\ &- \sin 2\alpha \sin \omega \cos \omega \tan \delta \\ &= \tan \delta \sin 2\alpha\,[\sin (\varphi + \omega) \cos (\varphi + \omega) - \sin \omega \cos \omega] \\ &= \tan \delta \sin 2\alpha \left[\frac{\sin 2(\varphi + \omega) - \sin 2\omega}{2}\right] \\ &= \underline{\tan \delta \sin 2\alpha \sin \varphi \cos (2\omega + \varphi);} \end{aligned}$$

and hence equation (24*e*) becomes

$$\left.\begin{aligned} &+ \sin \varphi\,[\sin (2\omega + \varphi) \sin 2\alpha - \cos (2\omega + \varphi) \cos 2\alpha] \\ &\qquad\qquad - \sin \varphi \cos (2\omega + \varphi) \\ &+ \sin \varphi\,[\sin (2\omega + \varphi) \cos 2\alpha \\ &\qquad\qquad + \cos (2\omega + \varphi) \sin 2\alpha] \tan \delta \\ &+ \sin \varphi\,[\sin (2\omega + \varphi) \tan \delta] - 2 \cos^2 \omega \tan \delta \end{aligned}\right\} = 0, \quad (24f)$$

and

$$\tan \delta = \frac{\sin \phi\,[\sin (2\omega + \phi) \sin 2\alpha - \cos (2\omega + \phi) \cos 2\alpha] - \sin \phi \cos (2\omega + \phi)}{2 \cos^2 \omega - \sin \phi\,[\sin (2\omega + \phi) \cos 2\alpha + \cos (2\omega + \phi) \sin 2\alpha] - \sin \phi \sin (2\omega + \phi)}.$$

By making $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ and $\cos 2\alpha = 1 - 2 \sin^2 \alpha$ in the numerator, and $\cos 2\alpha = 2 \cos \alpha \cos \alpha - 1$ and $\sin 2\alpha = 2 \sin \alpha \cos \alpha$ in the denominator, this becomes

$$\tan \delta = \frac{\sin \phi\,[\sin (2\omega + \phi)\, 2 \sin \alpha \cos \alpha - \cos (2\omega + \phi) + \cos (2\omega + \phi)\, 2 \sin^2 \alpha] - \sin \phi \cos (2\omega + \phi)}{2 \cos^2 \omega - \sin \phi\,[\sin (2\omega + \phi)\, 2 \cos^2 \alpha - \sin (2\omega + \phi) + \cos (2\omega + \phi)\, 2 \sin \alpha \cos \alpha] - \sin \phi \sin (2\omega + \phi)},$$

or

$$\tan \delta = \frac{2 \sin \phi \sin \alpha\,[\sin (2\omega + \phi) \cos \alpha + \cos (2\omega + \phi) \sin \alpha] - 2 \sin \phi \cos (2\omega + \phi)}{2 \cos^2 \omega - 2 \sin \phi \cos \alpha\,[\sin (2\omega + \phi) \cos \alpha + \cos (2\omega + \phi) \sin \alpha]},$$

which reduces to

$$\tan\delta = \frac{\sin\varphi\sin\alpha\sin(2\omega+\varphi+\alpha) - \sin\varphi\cos(2\omega+\varphi)}{\cos^2\omega - \sin\varphi\cos\alpha\sin(2\omega+\varphi+\alpha)}. \qquad (24g)$$

Equating this value of $\tan\delta$ with that in equation (23),

$$\frac{\sin\varphi\sin\alpha\sin(2\omega+\varphi+\alpha) - \sin\varphi\cos(2\omega+\varphi)}{\cos^2\omega - \sin\varphi\cos\alpha\sin(2\omega+\varphi+\alpha)} = \frac{\sin\varphi\sin 2\alpha}{\sin(2\omega+\varphi) - \sin\varphi\cos 2\alpha}.$$

Dividing by $\sin\varphi$, clearing of fractions and dividing by $\sin\alpha$, also transposing, this becomes

$$\left.\begin{aligned}
&\sin(2\omega+\varphi+\alpha)\sin(2\omega+\varphi)\\
&-\sin(2\omega+\varphi+\alpha)\sin\varphi\cos 2\alpha - \frac{\sin 2\alpha}{\sin\alpha}\cos^2\omega\\
&+\frac{\sin 2\alpha}{\sin\alpha}\cos\alpha\sin(2\omega+\varphi+\alpha)\sin\varphi\\
&-\frac{\cos(2\omega+\varphi)[\sin(2\omega+\varphi)-\sin\varphi\cos 2\alpha]}{\sin\alpha}
\end{aligned}\right\} = 0,$$

or

$$\left.\begin{aligned}
&\sin(2\omega+\varphi+\alpha)\sin(2\omega+\varphi)\\
&-\sin\varphi\cos 2\alpha\sin(2\omega+\varphi+\alpha) - 2\cos\alpha\cos^2\omega\\
&+\sin\varphi\, 2\cos^2\alpha\sin(2\omega+\varphi+\alpha)\\
&-\frac{\cos(2\omega+\varphi)[\sin(2\omega+\varphi)-\sin\varphi\cos 2\alpha]}{\sin\alpha}
\end{aligned}\right\} = 0.$$

Since $\qquad 2\cos^2\alpha - \cos 2\alpha = 1,$

this becomes

$$\sin(2\omega + \varphi + \alpha)\,[\sin(2\omega + \varphi) + \sin\varphi] - 2\cos\alpha\cos^2\omega - D = 0,$$

in which

$$D = \frac{\cos(2\omega + \varphi)\,[\sin(2\omega + \varphi) - \sin\varphi\cos 2\alpha]}{\sin\alpha},$$

or

$$\sin(2\omega+\varphi+\alpha)[2\sin(\omega+\varphi)\cos\omega] - 2\cos\alpha\cos^2\omega - D = 0,$$

or

$$\sin(2\omega+\varphi+\alpha)\sin(\omega+\varphi) - \cos\alpha\cos\omega - \frac{D}{2\cos\omega} = 0. \quad (25)$$

The formulæ for ω, δ, and E can now be found in the simplest manner. Equation (25) is satisfied for $2\omega+\varphi=90°$. Hence,

$$\omega = 45° - \frac{\varphi}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Substituting this value in equation (23), it becomes

$$\tan\delta = \frac{\sin\varphi\sin 2\alpha}{\sin(90-\varphi+\varphi) - \sin\varphi\cos 2\alpha}$$
$$= \frac{\sin\varphi\sin 2\alpha}{1 - \sin\varphi\cos 2\alpha}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

or the equivalent, but more convenient expression for calculation,

$$\tan(\delta + \alpha) = \frac{\tan\alpha}{\tan^2\left(45° - \frac{\varphi}{2}\right)}. \quad . \quad . \quad . \quad . \quad (28)$$

If, finally, $\omega = 45° - \frac{\varphi}{2}$ in equation (10), it becomes, remembering that $k^2 = \frac{h^2}{\cos^2\omega}$,

$$E = \frac{\cos^2\left(\varphi + 45° - \frac{\varphi}{2}\right)}{\cos(\alpha+\delta)} \cdot \frac{h^2\gamma}{2\cos^2\left(45° - \frac{\varphi}{2}\right)}$$

$$= \frac{\cos^2\left(45° + \frac{\varphi}{2}\right)}{\cos^2\left(45° - \frac{\varphi}{2}\right)} \cdot \frac{h^2\gamma}{2\cos(\alpha+\delta)}$$

$$= \frac{\sin^2\left[90° - \left(45° + \frac{\varphi}{2}\right)\right]}{\cos^2\left(45° - \frac{\varphi}{2}\right)} \cdot \frac{h^2\gamma}{2\cos(\alpha+\delta)};$$

hence $$E = \tan^2\left(45° - \frac{\varphi}{2}\right)\frac{h^2\gamma}{2\cos(\alpha+\delta)}, \quad . \quad . \quad . \quad (29)$$

or, from equation (28),

$$E = \frac{\tan\alpha}{\sin(\alpha+\delta)}\,\frac{h^2\gamma}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29a)$$

This last expression, however, when $\alpha = 0$ takes the indeterminate form $\frac{0}{0}$.

The earth-pressure upon a portion of the wall reaching from the depth h_0 to the depth $H = h_0 + h_1$ may be found

from equation (29) by substituting $H^2 - h_0^2$ in place of h^2, as is evident from the following:

Suppose the wall to have a height H, then $E_0 = C_0 \frac{H^2}{2}\gamma$, and likewise for a height h_0

$$E_1 = C_0 \frac{h_0^2}{2}\gamma \therefore E = E_0 - E_1 = C_0 \frac{H^2 - h_0^2}{2}\gamma, \quad . \quad . \quad (29b)$$

C_0 representing the constant quantity.

From equation (29b) $E = C(H^2 - h_0^2)$; hence $dE = 2CHdH - 2Ch_0dh_0$. Now let x equal the distance of the centre of pressure below the top of the wall, then

$$Ex = 2C\int_0^H H^2dH - 2C\int_0^h h_0^2dh,$$

or
$$C(H^2 - h_0^2)x = \tfrac{2}{3}CH^3 - \tfrac{2}{3}Ch_0^3,$$

or
$$x = \frac{2}{3}\frac{H^3 - h_0^3}{H^2 - h_0^2};$$

and if $y =$ the distance from bottom,

$$y = \frac{1}{3}\frac{H^3 - 3Hh_0^2 + 2h_0^3}{H^2 - h_0^2}. \quad . \quad . \quad . \quad . \quad (30)$$

Equation (30) holds good when the earth-surface is loaded and the loading is equal to a distributed load of the height h_0. Still, even then, h_0 is often so small that $\frac{h}{3}$ can be substituted for it just as for unloaded earth-surface.

In all cases δ is determined by equation (28).

Instead of using equations (28) and (29), the following simple construction can be used:

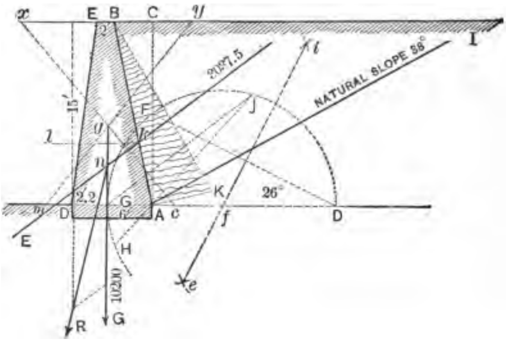


FIG. 4.

Draw (Fig. 4) AC and AD vertically and horizontally, each equal to h, also DF making the angle $FDG = 45° - \frac{\varphi}{2}$ with the horizontal. Through the points D and F describe a circle whose centre lies in AD. Then draw GH parallel to AB, and through A the straight line HJ. Then JG is the direction of the earth-pressure upon the wall AB. If AK is made perpendicular to AB, and equal to AH, then the ΔABK gives the intensity and distribution of the earth-pressure, or

$$E = \gamma \Delta ABK.$$

The proof of this construction is as follows: Conceive, in Fig. 4, JD and FG drawn, then

$$\tan AHG = \frac{AP}{PH} = \frac{AG \cos \alpha}{HG - [AG \sin \alpha = PG]};$$

in which AP represents the perpendicular let fall from A upon GH.

but $$AG : AF :: AF : AD = h,$$

therefore $$AG = \frac{\overline{AF}^2}{h} = h \tan^2\left(45° - \frac{\varphi}{2}\right).$$

Now

$$HG = GD \sin \alpha = (AG + AD) \sin \alpha$$
$$= h \sin \alpha + h \tan^2\left(45° - \frac{\varphi}{2}\right) \sin \alpha;$$

$\tan AHG =$

$$\frac{h \tan^2\left(45° - \frac{\varphi}{2}\right) \cos \alpha}{h \sin \alpha + h \tan^2\left(45° - \frac{\varphi}{2}\right) \sin \alpha - h \tan^2\left(45° - \frac{\varphi}{2}\right) \sin \alpha};$$

therefore

$$\tan AHG = \frac{\cos \alpha}{\sin \alpha} \tan^2\left(45° - \frac{\varphi}{2}\right) = \cot \alpha \tan^2\left(45° - \frac{\varphi}{2}\right).$$

From Fig. 4, $< GDJ = < AHG$, $< GDJ + < JGD = 90°$, and therefore

$$\tan JGD = \cot AHG = \tan \alpha \cot^2\left(45° - \frac{\varphi}{2}\right) = \tan (\alpha + \delta),$$

or $< JGD$ is the angle of the earth-pressure to the horizon.

Since, now, $< AHG = 90° - \alpha - \delta$,

$$AH = \frac{\cos \alpha}{\cos (\alpha + \delta)} AG = h \tan^2\left(45° - \frac{\varphi}{2}\right) \frac{\cos \alpha}{\cos (\alpha + \delta)},$$

and

$$\tfrac{1}{2} AH \,.\, AB = \tan^2\left(45° - \frac{\varphi}{2}\right) \frac{h^2}{2 \cos (\alpha + \delta)} = \frac{E}{\gamma}.$$

For a vertical wall the construction becomes much simpler. Draw, in Fig. 5, $AD = h$ horizontally, then DF making the angle $45° - \frac{\varphi}{2}$ with AD. Draw through D and F a circle with centre in DA and continue it around to K.

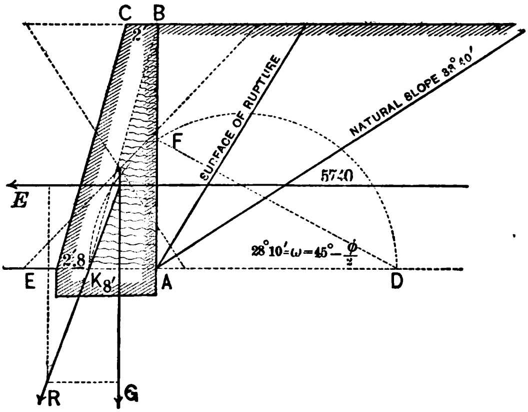


FIG. 5.

then the ΔABK gives the intensity and distribution of the earth-pressure, while in direction it is horizontal.

Hence $$E = \gamma \, \Delta ABK.$$

The proof is as follows (Fig. 5):

$$AK = \frac{AF^2}{AD} = \frac{h^2 \tan^2\left(45° - \frac{\varphi}{2}\right)}{h} = h \tan^2\left(45° - \frac{\varphi}{2}\right)$$

$$\tfrac{1}{2}AB \,.\, AK = \frac{h^2}{2} \tan^2\left(45° - \frac{\varphi}{2}\right) = \frac{E}{\gamma}.$$

As $\alpha = 0$, equation (28) gives $\tan \delta = 0$; $\therefore \delta = 0$ and E act normal to the surface of the wall.

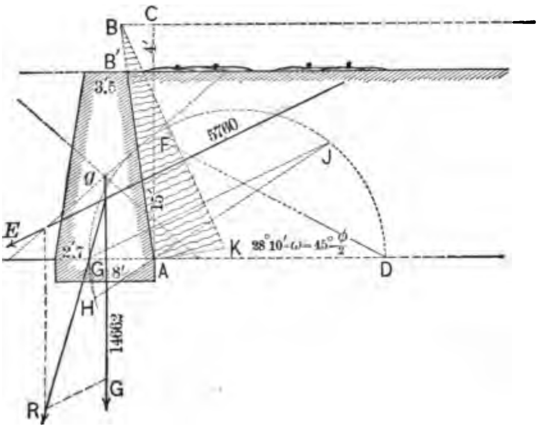


FIG. 6.

Finally, in Fig. 6 is the construction for loaded earth-surface. The point of application of the earth-pressure is always found by drawing through the centre of gravity of ΔABK a parallel to AK and producing it to meet the wall. The proof for this construction is the same as that for Fig. 4.

IV.

EARTH SURFACE PARALLEL TO SURFACE OF REPOSE.

$$\varepsilon = \varphi.$$

For this case,

$$E = \frac{\cos^2(\varphi - \alpha)}{\cos(\alpha + \delta)} \frac{l^2 \gamma}{2} = \left[\frac{\cos(\varphi - \alpha)}{\cos \alpha}\right]^2 \frac{h^2 \gamma}{2 \cos(\alpha + \delta)}; \quad (20)$$

a formula which holds good for all values of δ, and which for $\delta = 0$ or φ gives results usually accepted in previous theories of retaining-walls. In order to find the proper values of δ and ω, equations (16b) and (22b) must be used.

In equation (22b) replace $\sin(\varphi + \omega + \alpha + \delta)$ by $\sin(\varphi + \omega + \alpha) \cos \delta + \cos(\varphi + \omega + \alpha) \sin \delta$, and making $\varepsilon = \varphi$ it becomes

$$\left.\begin{array}{l} + \cos(\varphi + \omega) \cos(\varphi + \omega) \cos \delta \\ \qquad - \cos(\varphi + \omega) \sin(\varphi + \omega + \alpha) \cos \delta \sin \alpha \\ - \cos(\varphi + \omega) \cos(\varphi + \omega + \alpha) \sin \delta \sin \alpha \end{array}\right\} =$$

$$= \left\{\begin{array}{l} + \cos(\alpha - \varphi) \cos(\alpha + \delta) \cos \varphi \\ \qquad - \cos(\alpha - \varphi) \sin(\varphi + \omega + \alpha) \sin \omega \cos \delta \\ - \cos(\alpha - \varphi) \cos(\varphi + \omega + \alpha) \sin \delta \sin \omega; \end{array}\right.$$

dividing by $\cos\delta$ and transposing,

$$\left.\begin{array}{l} -\dfrac{\cos(\alpha-\varphi)\cos(\alpha+\delta)\cos\varphi}{\cos\delta} \\ \qquad\qquad +\cos(\alpha-\varphi)\sin(\varphi+\omega+\alpha)\sin\omega \\ +\cos(\varphi+\omega)\cos(\varphi+\omega) \\ \qquad\qquad -\cos(\varphi+\omega)\sin(\varphi+\omega+\alpha)\sin\alpha \end{array}\right\} =$$

$$= \left\{\begin{array}{l} +\cos(\varphi+\omega)\cos(\varphi+\omega+\alpha)\dfrac{\sin\delta}{\cos\delta}\sin\alpha \\ -\cos(\alpha-\varphi)\cos(\varphi+\omega+\alpha)\dfrac{\sin\delta}{\cos\delta}\sin\omega. \end{array}\right.$$

Since

$$-\frac{\cos(\alpha-\phi)\cos(\alpha+\delta)\cos\phi}{\cos\delta} = -\frac{\cos(\alpha-\phi)\cos\phi(\cos\alpha\cos\delta-\sin\alpha\sin\delta)}{\cos\delta}$$

$$= -\cos(\alpha-\phi)\cos\phi\cos\alpha+\cos(\alpha-\phi)\sin\alpha\frac{\sin\delta}{\cos\delta}\cos\phi,$$

the above expression reduces to

$$\tan\delta = \frac{\cos\alpha\cos(\alpha-\phi)\cos\phi-\cos\alpha\cos(\phi+\omega)\cos(\phi+\omega+\alpha)-\cos(\alpha-\phi)\sin\omega\sin(\phi+\omega+\alpha)}{\sin\alpha\cos(\alpha-\phi)\cos\phi-\sin\alpha\cos(\phi+\omega)\cos(\phi+\omega+\alpha)+\cos(\alpha-\phi)\sin\omega\cos(\phi+\omega+\alpha)}$$

and this equation fulfils the condition that the *sum of the moments of G, E, and R shall be zero.*

If equation ($16b$) is treated in a like manner, the resulting equation will fulfil the condition that the *sum of the forces parallel to the surface of rupture shall equal zero.* Making $\varepsilon = \varphi$ in equation ($16b$), it reduces to

$$\sin(\alpha+\omega)\cos(\varphi+\omega)\cos(\alpha+\delta) \\ -\sin(\varphi+\alpha+\omega+\delta)\cos(\varphi+\omega)\cos(\alpha-\varphi) = 0,$$

or

$$\sin(\alpha+\omega)\cos(\alpha+\delta)-\sin(\varphi+\omega+\alpha)\cos(\alpha-\varphi)\cos\delta \\ -\cos(\varphi+\omega+\alpha)\cos(\alpha-\varphi)\sin\delta=0,$$

or

$$\frac{\sin(\alpha+\omega)\cos\alpha\cos\delta}{\cos\delta}-\frac{\sin(\alpha+\omega)\sin\alpha\sin\delta}{\cos\delta}- \\ \sin(\varphi+\omega+\alpha)\cos(\alpha-\varphi)-\frac{\cos(\varphi+\omega+\alpha)\cos(\alpha-\varphi)\sin\delta}{\cos\delta}=0;$$

therefore

$$\tan\delta=\frac{\cos\alpha\sin(\alpha+\omega)-\sin(\varphi+\omega+\alpha)\cos(\alpha-\varphi)}{\sin(\alpha+\omega)\sin\alpha+\cos(\varphi+\omega+\alpha)\cos(\alpha-\varphi)}.$$

Setting both values of $\tan\delta$ equal to each other and clearing of fractions, the following expression is obtained:

$$\begin{aligned}
&+\cos\alpha\cos\varphi\sin\alpha\sin(\omega+\alpha)\cos(\alpha-\varphi)\\
&-\cos\alpha\sin\alpha\sin(\omega+\alpha)\cos(\omega+\varphi)\cos(\omega+\varphi+\alpha)\\
&-\sin\omega\sin\alpha\sin(\omega+\alpha)\cos(\alpha-\varphi)\sin(\varphi+\omega+\alpha)\\
&+\cos\alpha\cos\varphi\cos(\alpha-\varphi)\cos(\varphi+\omega+\alpha)\cos(\alpha-\varphi)\\
&-\cos\alpha\cos(\varphi+\omega)\cos^2(\varphi+\omega+\alpha)\cos(\alpha-\varphi)\\
&-\sin\omega\cos^2(\alpha-\varphi)\sin(\varphi+\omega+\alpha)\cos(\varphi+\omega+\alpha)
\end{aligned}$$

for the first member of the equation, and

$$\begin{aligned}
&+\cos\alpha\cos\varphi\sin\alpha\sin(\omega+\alpha)\cos(\alpha+\varphi)\\
&-\sin\alpha\cos\alpha\sin(\omega+\alpha)\cos(\omega+-)\cos(\varphi+\omega+\alpha)\\
&+\sin\omega\cos\alpha\sin(\omega+\alpha)\cos(\alpha-\varphi)\cos(\varphi+\omega+\alpha)\\
&-\sin\alpha\cos\varphi\cos^2(\alpha-\varphi)\sin(\varphi+\omega+\alpha)\\
&+\sin\alpha\cos(\varphi+\omega)\cos(\varphi+\omega+\alpha)\cos(\alpha-\varphi)\sin(\varphi+\omega+\alpha)\\
&-\sin\omega\cos^2(\alpha-\varphi)\cos(\varphi+\omega+\alpha)\sin(\varphi+\omega+\alpha)
\end{aligned}$$

for the second member.

The first terms, second terms, and sixth terms cancel. Divide the equation by $\cos(\alpha-\varphi)$. Terms number 3 combined give

$$-\sin\omega\sin(\omega+\alpha)\,[\sin\alpha\sin(\phi+\omega+\alpha)+\cos\alpha\cos(\phi+\omega+\alpha)],$$

which becomes

$$-\sin\omega\sin(\omega+\alpha)\cos(\varphi+\omega).$$

Terms number 5 combined give

$$-\cos(\phi+\omega)\cos(\phi+\omega+\alpha)\,[\cos\alpha\cos(\phi+\omega+\alpha)+\sin\alpha\sin(\phi+\omega+\alpha)],$$

which becomes

$$-\cos(\varphi+\omega+\alpha)\cos(\varphi+\omega)\cos(\varphi+\omega).$$

Terms number 4 combined give

$$+\cos\varphi\cos(\alpha-\varphi)\,[\cos\alpha\cos(\varphi+\omega+\alpha)+\sin\alpha\sin(\varphi+\omega+\alpha)],$$

which becomes

$$+\cos\varphi\cos(\alpha-\varphi)\cos(\varphi+\omega),$$

and hence, after dividing by $\cos(\varphi+\omega)$, the equation above reduces to

$$\cos(\alpha-\varphi)\cos\varphi-\cos(\varphi+\omega+\alpha)\cos(\varphi+\omega)-\sin(\omega+\alpha)\sin\omega=0, \quad (31)$$

and this equation is fulfilled for

$$\omega=90^\circ-\varphi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

In order to find that value of δ which satisfies all conditions of equilibrium, substitute the above value of ω in the first expression for $\tan\delta$ and obtain $\frac{0}{0}$. If, according to

the method for discussing indeterminate fractions, the first differentials of the numerator and denominator and their ratio are found, and ω made equal to $90° - \varphi$, the value of $\tan \delta$ will be found.

The differential of the numerator is

$$d[-\cos\alpha\cos(\varphi+\omega)\cos(\varphi+\omega+\alpha)-\cos(\alpha-\varphi)\sin\omega\sin(\varphi+\omega+\alpha)],$$

which equals

$$\left\{\begin{array}{l} +\cos\alpha\cos(\varphi+\omega+\alpha)\sin(\varphi+\omega) \\ +\cos\alpha\cos(\varphi+\omega)\sin(\varphi+\omega+\alpha) \\ -\cos(\alpha-\varphi)\sin(\varphi+\omega+\alpha)\cos\omega \\ -\cos(\alpha-\varphi)\sin\omega\cos(\varphi+\omega+\alpha) \end{array}\right\} d\omega.$$

Substituting for ω, $90° - \varphi$, this becomes

$$\left\{\begin{array}{l} +\cos\alpha\cos(\varphi+90°-\varphi+\alpha)\sin(\varphi+90°-\varphi) \\ +\cos\alpha\cos(\varphi+90°-\varphi)\sin(\varphi+90°-\varphi+\alpha) \\ -\cos(\alpha-\varphi)\sin(\varphi+90°-\varphi+\alpha)\cos(90°-\varphi) \\ +\cos(\alpha-\varphi)\sin(90°-\varphi)\cos(\varphi+90°-\varphi+\alpha) \end{array}\right\} d\omega.$$

As the second term reduces to zero, this becomes

$$[\cos\alpha\sin\alpha - \cos(\alpha-\varphi)\cos\alpha\sin\varphi + \cos(\alpha-\varphi)\cos\varphi\sin\alpha]\, d\omega,$$

or

$$\left[\frac{\sin 2\alpha}{2} - \cos(\alpha-\varphi)(\cos\alpha\sin\varphi - \cos\varphi\sin\alpha)\right] d\omega,$$

or

$$\left[\frac{\sin 2\alpha}{2} - \cos(\alpha-\varphi)\sin(\varphi-\alpha)\right] d\omega$$

$$= \left[\frac{\sin 2\alpha}{2} + \frac{\sin 2(\varphi-\alpha)}{2}\right] d\omega,$$

or

$$\left[\frac{2 \sin \frac{1}{2}(2\varphi - 2\alpha + 2\alpha) \cos \frac{1}{2}(2\varphi - 2\alpha - 2\alpha)}{2}\right] d\omega,$$

which equals $\sin \varphi \cos(\varphi - 2\alpha)\, d\omega.$

The differential of the denominator is

$$\left\{\begin{array}{l} + \sin \alpha \cos(\varphi + \omega + \alpha) \sin(\varphi + \omega) \\ + \sin \alpha \cos(\varphi + \omega) \sin(\varphi + \omega + \alpha) \\ + \cos(\alpha - \varphi) \cos(\varphi + \omega + \alpha) \cos \omega \\ + \cos(\alpha - \varphi) \sin \omega \sin(\varphi + \omega + \alpha) \end{array}\right\} d\omega.$$

Substituting $90° - \varphi$ for ω, and this becomes

$$[\sin \alpha \sin \alpha + \cos(\alpha - \varphi) \sin \alpha \sin \varphi + \cos(\alpha - \varphi) \cos \varphi \cos \alpha]\, d\omega,$$

or

$$[\sin^2 \alpha + \cos(\alpha - \varphi)(\sin \varphi \sin \alpha + \cos \varphi \cos \alpha)] d\omega,$$

or

$$[1 - \cos^2 \alpha + \cos(\alpha - \varphi) \cos(\alpha - \varphi)] d\omega$$
$$= \left[1 - \frac{\cos 2\alpha}{2} - \frac{1}{2} + \frac{\cos 2(\alpha - \varphi)}{2} + \frac{1}{2}\right] d\omega,$$

or $[1 - \sin \varphi \sin(\varphi - 2\alpha)] d\omega;$

therefore

$$\tan \delta = \frac{\sin \varphi \cos(\varphi - 2\alpha)}{1 - \sin \varphi \sin(\varphi - 2\alpha)}. \quad . \quad . \quad (33)$$

To find an expression for the sin δ, clear equation (33)

of fractions and deduce $\tan\delta - \tan\delta \sin\varphi \sin(\varphi - 2\alpha) = \sin\varphi \cos(\varphi - 2\alpha)$. Multiplying by $\cos\delta$,

$$\sin\delta - \sin\delta \sin\varphi \sin(\varphi - 2\alpha) = \sin\varphi \cos(\varphi - 2\alpha) \cos\delta,$$

or

$$\sin\delta = \sin\varphi [\sin\delta \sin(\varphi - 2\alpha) + \cos(\varphi - 2\alpha) \cos\delta];$$

therefore

$$\sin\delta = \sin\varphi \cos(2\alpha - \varphi + \delta), \quad . \quad . \quad (34)$$

from which the results of III. can be deduced.

If the earth-surface is parallel to the surface of repose, or makes the angle φ with the horizontal, then, under the assumption of a plane surface of rupture, $\delta = \varphi$ only when the wall is vertical (make $\alpha = 0$ in equation (33), then $\tan\delta = \tan\varphi$; $\therefore \delta = \varphi$), and $\delta = 0$ only when the angle of the wall with the vertical $\alpha = 45° + \frac{\varphi}{2}$.

As it is often more convenient in determining the direction of the earth-pressure to know the angle $(\alpha + \delta)$ of E with the horizon, $\tan(\alpha + \delta)$ may be expressed in terms of $\tan\alpha$ and $\tan\delta$, remembering that

$$\cos\alpha - \sin\varphi \sin(\varphi - \alpha) = \cos\varphi \cos(\varphi - \alpha),$$

and hence

$$\tan(\alpha + \delta) = \frac{\sin\alpha + \sin\varphi \cos(\varphi - \alpha)}{\cos\varphi \cos(\varphi - \alpha)}. \quad . \quad (34a)$$

With reference to a limited portion of wall which does

not reach as far as the surface, and with reference to loaded earth-surface, the same remarks hold good as in III.

Instead of formulæ (20) and (33) or (34), the following construction may be used:

Draw through A, Fig. 7, a parallel to the earth-surface,

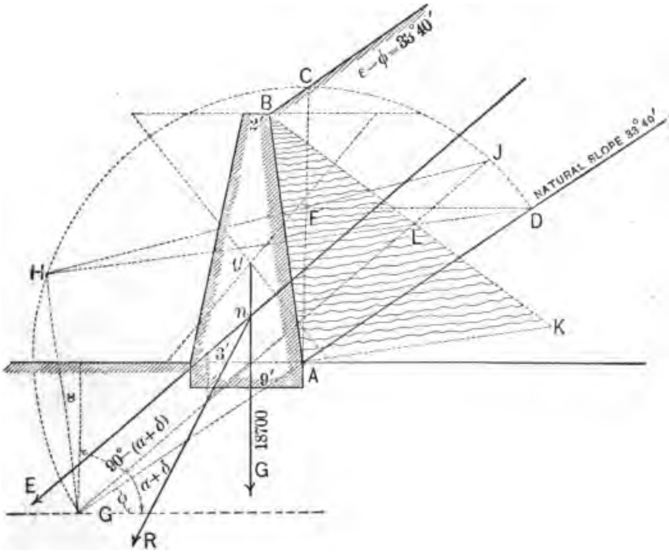


FIG. 7.

and with AC as a radius describe the circle ADG. Draw DF horizontal and GH parallel to AB, and then the straight line HFJ. Then the direction of the earth-pressure is GJ; and if AK is made perpendicular to AB and equal to HF, $E = \gamma \Delta ABK$, and the triangle gives the distribution of the pressure. The point of application is found by drawing through the centre of gravity of the triangle a perpendicular to AB.

The proof of this construction is as follows:

Conceive HD drawn, and its intersection with GJ to be at L. Then from the notation of Fig. 3, where $\varepsilon = \varphi$,

$$FD = AD \cos \varphi, \qquad HD = 2AD \cos (\varphi - \alpha).$$

Since, now, $\angle JLD = \angle JHD + \varphi - \alpha$, by expressing $\tan JLD$ by tan of JHD and $\varphi - \alpha$, after reducing,

$$\tan JLD = \frac{\cos \varphi \sin (2\alpha - \varphi) + \sin 2(\varphi - \alpha)}{1 + \cos 2(\varphi - \alpha) - \cos \varphi \cos (2\alpha - \varphi)},$$

or

$$\tan JLD = \frac{\sin \varphi \cos (\varphi - 2\alpha)}{1 + \sin \varphi \sin (\varphi - 2\alpha)} = \tan \delta.$$

Since HD is perpendicular to AB, the earth-pressure has the direction GJ. Further,

$$HF = \frac{FD \sin \alpha}{\sin (\alpha + \delta - \varphi)} = \frac{\sin \alpha \cos \varphi}{\sin (\alpha + \delta - \varphi)} AD,$$

$AD = \dfrac{l \cos (\varphi - \alpha)}{\cos \varphi}$, or, with reference to the value of FD.

$\Delta ABK = \dfrac{\cos (\varphi - \alpha) \sin \alpha}{\sin (\alpha + \delta - \varphi)} \dfrac{l^2}{2}$, and since from equation (34) $\sin (\alpha + \delta - \varphi) \cos (\varphi - \alpha) = \sin \alpha \cos (\alpha + \delta)$,

$$\Delta ABK = \frac{\cos^2 (\varphi - \alpha)}{\cos (\alpha + \delta)} \frac{l^2}{2} = \frac{E}{\gamma}.$$

RECAPITULATION OF FORMULÆ.

Inclined earth-surface, plane:

$$n = \sqrt{\frac{\sin(\varphi + \delta)\sin(\varphi - \varepsilon)}{\cos(\alpha + \delta)\cos(\alpha - \varepsilon)}}. \quad . \quad . \quad . \quad (18)$$

The tan δ deduced from formulæ (22*b*) and (16*b*):

$$\tan\delta = \frac{\sin(2\alpha - \varepsilon) - K\sin 2(\alpha - \varepsilon)}{K - \cos(2\alpha - \varepsilon) + K\cos 2(\alpha - \varepsilon)},$$

in which

$$K = \frac{\cos\varepsilon - \sqrt{\cos^2\varepsilon - \cos^2\varphi}}{\cos^2\varphi},$$

$$E = \left[\frac{\cos(\varphi - \alpha)}{(n + 1)\cos\alpha}\right]^2 \frac{h^2\gamma}{2\cos(\alpha + \delta)}. \quad . \quad . \quad . \quad (19)$$

Earth-surface parallel to natural slope:

$$\varepsilon = \varphi;$$

$$E = \left[\frac{\cos(\varphi - \alpha)}{\cos\alpha}\right]^2 \frac{h^2\gamma}{2\cos(\alpha + \delta)}; \quad . \quad . \quad . \quad (20)$$

$$\omega = 90^\circ - \varphi; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

$$\tan(\alpha + \delta) = \frac{\sin\alpha + \sin\varphi\cos(\varphi - \alpha)}{\cos\varphi\cos(\varphi - \alpha)}; \quad . \quad . \quad . \quad (34a)$$

$$\tan\delta = \frac{\sin\varphi\cos(\varphi - 2\alpha)}{1 - \sin\varphi\sin(\varphi - 2\alpha)}. \quad . \quad . \quad . \quad . \quad (33)$$

Horizontal earth-surface:

$$\omega = 45° - \frac{\varphi}{2}; \quad \dots \dots \dots \quad (26)$$

$$\tan \delta = \frac{\sin \varphi \sin 2\alpha}{1 - \sin \varphi \cos 2\alpha}; \quad \dots \dots \quad (27)$$

$$\tan (\alpha + \delta) = \frac{\tan \alpha}{\tan^2 \left(45° - \frac{\varphi}{2}\right)}; \quad \dots \dots \quad (28)$$

$$E = \tan^2 \left(45° - \frac{\varphi}{2}\right) \frac{h^2\gamma}{2 \cos (\alpha + \delta)}; \quad \dots \quad (29)$$

$$E = \frac{\tan \alpha}{\sin (\alpha + \delta)} \cdot \frac{h^2\gamma}{2}. \quad \dots \dots \dots \quad (29a)$$

If $\alpha = 0$, then $\delta = 0$, and

$$E = \tan^2 \left(45° - \frac{\varphi}{2}\right) \frac{h^2\gamma}{2}. \quad \dots \dots \quad (29d)$$

If $\alpha = \left(45° - \frac{\varphi}{2}\right) = \omega$, then $\delta = \varphi$, and

$$E = \frac{\tan \left(45° - \frac{\varphi}{2}\right)}{\sin \left(45° + \frac{\varphi}{2}\right)} \frac{h^2\gamma}{2}. \quad \dots \dots \quad (29e)$$

If the surface is loaded, substitute $H^2 + h'^2$ for h^2, or consider h to be the height of the earth increased by the height of an amount of earth weighing as much as the applied load.

NOMENCLATURE.

Height of wall	H
Thickness at base	b
Thickness at top	b'
Batter in inches per foot of H on front face ...	d
Weight per cubic foot	W
Total weight of wall	G
Angle of repose of earth	φ
Angle made by surface of rupture with vertical	ω
Weight of cubic foot of earth	γ
Total thrust of earth against wall	E
Angle made with the horizontal by the surface of the earth	ε
Angle made by rear face of wall with the vertical	α
Angle made with normal by E	δ
Dist. of point where the resultant pressure cuts the base from the front edge of the wall ..	q
The resultant pressure due to E and G	R

NOTE.

For the translation of Prof. Weyrauch's paper the writer is indebted to the labor of Prof. A. J. Du Bois, of the Sheffield Scientific School, Yale College, who had copies printed by the electric-pen process. However, only the leading equations of Prof. Weyrauch were given; hence a great deal of labor has been devoted to expanding, verifying, and filling in the intermediate steps of the work, and this nucleus of the mathematical part alone has grown to about double the original quantity.

M. A. H.

REFERENCES.

A brief outline of the theories advanced by the following writers can be found in "*Neue Theorie des Erddruckes,*" *Dr. E. Winkler, Wien,* 1872:

D'Antony,	Hoffmann,	Poncelet,
Andé,	Holzhey,	Prony,
Andoy,	de Lafont,	Rankine,
Belidor,	Levi,	Rebhann,
Blaveau,	de Köszegh Martony,	Rondelet,
Bullet,	Maschek,	Saint-Guilhem,
Considère,	Mayniel,	Saint-Venant,
Coulomb,	Mohr,	Sallonnier,
Couplet,	Montlong,	Scheffler,
Culmann,	Moseley,	Trincaux,
Français,	Navier,	Vauban,
Gadroy,	Ortmann,	Winkler,
Gauthey,	v. Ott,	Woltmann.
Hagen,	Persy,	

AUDÉ. Poussée des Terres. Nouvelles expériences sur la poussée des terres. Paris, 1849.

BAKER-CURIE. Note sur la brochure de M. B. Baker théorie. Annales des Ponts et Chaussées, pp. 558–592, 1882.

—— The actual lateral pressure of earthwork. Van Nostrand's Magazine, xxv, 1881; also Van Nostrand's Science Series, No. 56.

BOUSSINESQ. Complément à de précédentes notes sur la poussée des terres. *Annales P. et C., 1884.

BOUSIN. Equilibrium of pulverulent bodies. The equilibrium of earth when confined by a wall. †Van N., 1881.

CAIN. Modification of Weyrauch's Theory. Van N., 1880.

—— Earth-pressure. Modification of Weyrauch's Theory. Criticism of Baker's articles. Van N., 1882.

—— Uniform cross-section, and T abutments: their proper proportions and sizes, deduced from Rankine's general formulas. Van N., 1872.

—— Practical designing of retaining-walls. Van N. Science Series, No. 3, 1888.

CHAPERON. Observations sur le mémoire de M. de Sazilly (1851). Stabilité et consolidation des talus. Annales P. et C., 1853.

CONSIDÈRE. Note sur la poussée des terres. Annales P. et C., 1870.

COUSINERY. Détermination graphique de l'épaisseur des murs de soutènement. Annales P. et C., 1841.

DE LAFONT. Sur la poussée des terres et sur les dimensions à donner, suivant leurs profils, aux murs de soutènement et de reservoirs d'eau. Annales P. et C., 1866.

DE SAZILLY. Sur les conditions d'équilibre des massifs de terre, et sur les revêtements des talus. Annales P. et C., 1851.

EDDY. Retaining-walls treated graphically. Van N., 1877.

FLAMANT. Note sur la poussée des terres. Annales P. et C., 1882.

—— Résumé d'articles publiés par la Société des Ingénieures Civils de Londres sur la poussée des terres. Annales P. et C., 1883.

---

* Annales des Ponts et Chaussées.

† Van Nostrand's Magazine.

FLAMANT. Note sur la poussée des terres. Annales P. et C., 1872.

—— Mémoire sur la stabilité de la terre sans cohésion par W. J. Macquorm Rankine (Extrait 1856–57). Annales P. et C., 1874.

GOBIN. Détermination précis de la stabilité des murs de soutènement et de la poussée des terres. Annales P. et C., 1883.

GOULD. Theory of J. Dubosque. Van N., 1883.

—— Designing. Van N., 1877.

JACOB. Practical designing of retaining-walls. Van N., 1873; also Van N. Science Series, No. 3.

JACQUIER. Note sur la détermination graphique de la poussée des terres. Annales P. et C., 1882.

KLEITZ. Détermination de la poussée des terres et établissement des murs de soutènement. Annales P. et C., 1844.

LAGREUE. Note sur la poussée des terres avec ou sans surcharges. Annales P. et C.. 1881.

L'ÈVEILLÉ. De l'emploi des contre-forts. Annales P. et C. 1844.

LEYGUE. Sur les grands murs de soutènement de la ligne de Mezamet a Bédarieux. Annales P. et C., 1887.

—— Nouvelle recherche sur la poussée des terres et le profil de revêtement le plus économique. Annales P. et C., 1885.

MERRIMAN. On the theories of the lateral pressure of sand against retaining walls. (School of Mines Quarterly.) Engineering News, 1888.

—— The theory and calculation of earthwork. Engineering News, 1885.

REBHANN. Theorie des Erddruckes und der Futtermauern, Wien, 1870 and 1871,

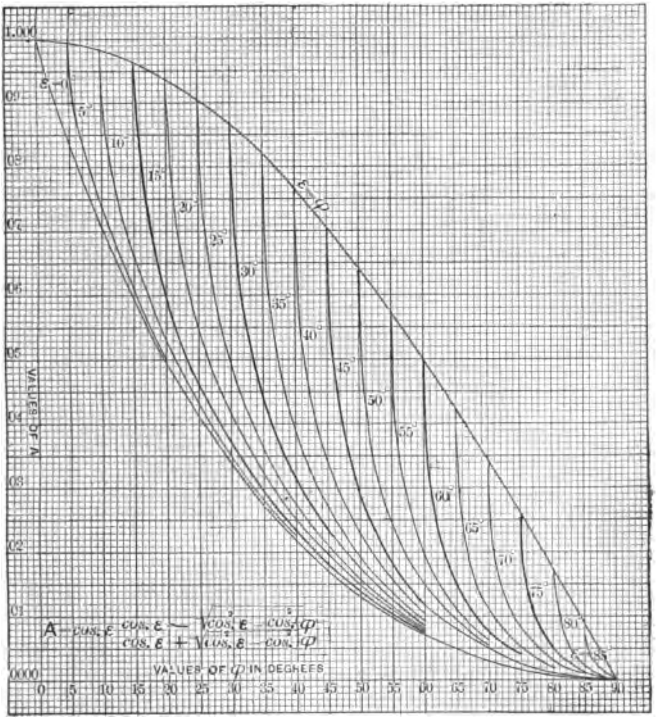
Saint-Guilhem. Sur la poussée des terres avec ou sans surcharge. Annales P. et C., 1858.

Scheffler-Fournie. Traité de la stabilité des constructions. Paris, 1864.

Tate. Surcharged and different forms of retaining-walls. Van N., 1873; also Van N. Science Series, No. 7
Also published by E. & F. N. Spon.

Thornton. Theory. Van N., 1879.

DIAGRAM I.



1.000
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VALUES OF A
VALUES OF φ IN DEGREES
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
$\varepsilon=0$
$\varepsilon=\varphi$
$A = \cos.\varepsilon \frac{\cos.\varepsilon - \sqrt{\cos.^2\varepsilon - \cos.^2\varphi}}{\cos.\varepsilon + \sqrt{\cos.^2\varepsilon - \cos.^2\varphi}}$

TABLES.

Table I contains the crushing-strengths and the average weights of stone likely to be used in the construction of retaining-walls and foundations; also the average weights of different earths.

Table II contains the coefficients of friction, limiting angles of friction, and the reciprocals of the coefficients of friction for various substances.

Tables III, IV, and V contain the values of the coefficients [see equation (1′)] (B), (C), (D) and (E), where

$$(B) = \frac{\cos(\epsilon - \alpha)}{\cos^2\alpha \cos\epsilon}, \quad (C) = \sin^2\alpha, \quad (D) = \left\{ \frac{\cos(\epsilon - \alpha)}{\cos\epsilon} \right\}^2$$

and $$(E) = 2 \sin\alpha \sin\epsilon \frac{\cos(\epsilon - \alpha)}{\cos\epsilon}.$$

The tables were computed with a Thacher calculating instrument and checked by means of diagrams. It is believed that they are correct to the second place of decimals; an error in the third place of decimals does not affect the results for practical purposes.

Table VI contains the natural sines, cosines and tangents.

TABLE I.

VALUES OF W.

Name of Substance.	Crushing Lds. in tons per sq. ft.	Average weight in lbs. per cu. ft.
Alabaster	....	144
Brick, best pressed	40 to 300	150
" common hard	....	125
" soft inferior	....	100
Chalk	20 to 30	156
Cement, loose	....	49.6 to 102
Flint	....	162
Feldspar	....	166
Granite	300 to 1200	170
Gneiss	....	168
Greenstone, trap	....	187
Hornblende, black	....	203
Limestones and Marbles, ordinary	250 to 1000	{ 164.4 { 168
Mortar, hardened	....	103
Quartz, common	....	165
Sandstone	150 to 550	151
Shales	....	162
Slate	400 to 800	175
Soapstone	....	170

VALUES OF γ.

Name of Substance.	Average weight in lbs. per cu. ft.
Earth, common loam, loose	72 to 80
" " " shaken	82 " 92
" " " rammed moderately	90 " 100
Gravel	90 " 106
Sand	90 " 106
Soft flowing mud	104 " 120
Sand perfectly wet	118 " 129

TABLE II.

* ANGLES AND COEFFICIENTS OF FRICTION.

	tan ϕ.	ϕ	$\frac{1}{\tan \phi}$
Dry masonry and brickwork	0.6 to 0.7	31° to 35°	1.67 to 1.43
Masonry and brickwork with damp mortar.......	0.74	$36\frac{1}{2}$°	1.35
Timber on stone...........	about 0.4	22°	2.5
Iron on stone	0.7 to 0.3	35° to $16\frac{2}{3}$°	1.43 to 3.33
Timber on timber..........	0.5 " 0.2	$26\frac{1}{2}$° " $11\frac{1}{3}$°	2 " 5
Timber on metals..........	0.6 " 0.2	31° " $11\frac{1}{3}$°	1.67 " 5
Metals on metals...........	0.25 " 0.15	14° " $8\frac{1}{2}$°	4 " 6.67
Masonry on dry clay.......	0.51	27°	1.96
" " moist clay.....	0.33	$18\frac{1}{4}$°	3.
Earth on earth	0.25 to 1.0	14° to 45°	4 to 1
Earth on earth, dry sand, clay, and mixed earth....	0.38 " 0.75	21° " 37°	2.63 " 1.33
Earth on earth, damp clay .	1.0	45°	1
Earth on earth, wet clay. .	0.31	17°	3.23
Earth on earth, shingle and gravel..................	0.81	39° to 48°	1.23 to 0.9

* From Rankine's Applied Mechanics.

TABLE III.

ϵ	$a = 5°$	$a = 6°$	$a = 7°$	$a = 8°$	$a = 9°$
	(*B*)	(*B*)	(*B*)	(*B*)	(*B*)
0	1.004	1.005	1.007	1.010	1.012
5	1.012	1.015	1.018	1.022	1.026
10	1.019	1.024	1.029	1.035	1.040
15	1.027	1.034	1.041	1.048	1.055
20	1.036	1.044	1.052	1.062	1.071
25	1.045	1.055	1.065	1.076	1.088
30	1.055	1.066	1.079	1.092	1.105
35	1.065	1.079	1.094	1.109	1.124
40	1.078	1.094	1.111	1.129	1.147
45	1.093	1 111	1.131	1.152	1.173
	(*C*)	(*C*)	(*C*)	(*C*)	(*C*)
	0.008	0.011	0.015	0.019	0.024

TABLE IV.

ϵ	$a = 5°$	$a = 6°$	$a = 7°$	$a = 8°$	$a = 9°$
	(*D*)	(*D*)	(*D*)	(*D*)	(*D*)
0	0.992	0.989	0.985	0.981	0.976
5	1.008	1.008	1.006	1.005	1.003
10	1.023	1.026	1.028	1.030	1.031
15	1.040	1.046	1.051	1.056	1.060
20	1.057	1.066	1.075	1.084	1.092
25	1.075	1.089	1.102	1.114	1.125
30	1.096	1.113	1.130	1.147	1.163
35	1.118	1.140	1.164	1.183	1.204
40	1.144	1.172	1.199	1.226	1.253
45	1.174	1.208	1.242	1.276	1.309

TABLE V.

ϵ	$a = 5°$	$a = 6°$	$a = 7°$	$a = 8°$	$a = 9°$
	(*E*)	(*E*)	(*E*)	(*E*)	(*E*)
0	0	0	0	0	0
5	0.015	0.018	0.021	0.024	0.027
10	0.031	0.037	0.043	0.049	0.055
15	0.046	0.055	0.065	0.074	0.083
20	0.061	0.074	0.086	0.099	0.112
25	0.076	0.092	0.108	0.124	0.140
30	0.091	0.110	0.130	0.149	0.169
35	0.106	0.128	0.151	0.174	0.197
40	0.120	0.145	0.172	0.198	0.225
45	0.134	0.162	0.192	0.222	0.253

TABLE III—*Continued.*

e	$a = 10°$	$a = 11°$	$a = 12°$	$a = 13°$	$a = 14°$
	(*B*)	(*B*)	(*B*)	(*B*)	(*B*)
0	1.015	1.019	1.022	1.026	1.031
5	1.031	1.037	1.041	1.047	1.053
10	1.046	1.055	1.061	1.068	1.076
15	1.063	1.073	1.081	1.090	1.100
20	1.081	1.092	1.103	1.112	1.125
25	1.099	1.112	1.124	1.136	1.150
30	1.119	1.135	1.151	1.163	1.179
35	1.141	1.159	1.175	1.195	1.211
40	1.166	1.186	1.205	1.225	1.245
45	1.195	1.218	1.240	1.263	1.288
	(*C*)	(*C*)	(*C*)	(*C*)	(*C*)
	0.030	0.036	0.043	0.051	0.029

TABLE IV—*Continued.*

e	$a = 10°$	$a = 11°$	$a = 12°$	$a = 13°$	$a = 14°$
	(*D*)	(*D*)	(*D*)	(*D*)	(*D*)
0	0.970	0.964	0.957	0.950	0.942
5	1.000	0.997	0.993	0.988	0.983
10	1.031	1.031	1.030	1.028	1.026
15	1.064	1.067	1.069	1.061	1.072
20	1.099	1.105	1.110	1.116	1.121
25	1.136	1.147	1.156	1.165	1.173
30	1.178	1.194	1.204	1.220	1.232
35	1.224	1.244	1.262	1.281	1.300
40	1.291	1.304	1.328	1.353	1.377
45	1.342	1.375	1.407	1.438	1.469

TABLE V—*Continued.*

e	$a = 10°$	$a = 11°$	$a = 12°$	$a = 13°$	$a = 14°$
	(*E*)	(*E*)	(*E*)	(*E*)	(*E*)
0	0	0	0	0	0
5	0.030	0.032	0.036	0.039	0.042
10	0.061	0.067	0.073	0.079	0.085
15	0.093	0.102	0.111	0.119	0.130
20	0.124	0.137	0.150	0.163	0.175
25	0.156	0.173	0.189	0.205	0.221
30	0.188	0.208	0.216	0.248	0.269
35	0.220	0.244	0.268	0.292	0.316
40	0.252	0.280	0.308	0.336	0.365
45	0.284	0.316	0.349	0.382	0.415

TABLE III—*Continued.*

ϵ	$a = 15°$	$a = 16°$	$a = 17°$	$a = 18°$	$a = 20°$
	(*B*)	(*B*)	(*B*)	(*B*)	(*B*)
0	1.035	1.040	1.048	1.051	1.062
5	1.059	1.066	1.076	1.081	1.098
10	1.084	1.093	1.104	1.112	1.132
15	1.110	1.120	1.134	1.138	1.168
20	1.135	1.149	1.165	1.177	1.218
25	1.165	1.179	1.197	1.211	1.245
30	1.195	1.212	1.233	1.248	1.288
35	1.229	1.249	1.272	1.291	1.339
40	1.268	1.291	1.317	1.340	1.389
45	1.313	1.338	1.369	1.393	1.451
	(*C*)	(*C*)	(*C*)	(*C*)	(*C*)
	0.067	0.076	0.086	0.095	0.117

TABLE IV—*Continued.*

ϵ	$a = 15°$	$a = 16°$	$a = 17°$	$a = 18°$	$a = 20°$
	(*D*)	(*D*)	(*D*)	(*D*)	(*D*)
0	0.933	0.924	0.915	0.905	0.883
5	0.977	0.971	0.964	0.957	0.940
10	1.023	1.018	1.016	1.011	1.000
15	1.072	1.073	1.071	1.069	1.068
20	1.124	1.127	1.129	1.131	1.132
25	1.181	1.188	1.194	1.200	1.208
30	1.244	1.256	1.266	1.276	1.293
35	1.316	1.332	1.348	1.363	1.390
40	1.400	1.422	1.444	1.465	1.505
45	1.500	1.530	1.559	1.588	1.643

TABLE V—*Continued.*

ϵ	$a = 15°$	$a = 16°$	$a = 17°$	$a = 18°$	$a = 20°$
	(*E*)	(*E*)	(*E*)	(*E*)	(*E*)
0	0	0	0	0	0
5	0.045	0.047	0.050	0.053	0.058
10	0.091	0.097	0.102	0.108	0.119
15	0.139	0.148	0.157	0.165	0.183
20	0.188	0.200	0.213	0.225	0.249
25	0.238	0.254	0.270	0.177	0.318
30	0.289	0.309	0.329	0.349	0.389
35	0.341	0.365	0.390	0.414	0.463
40	0.394	0.423	0.452	0.481	0.539
45	0.448	0.482	0.516	0.551	0.620

Table VI.

NATURAL SINES, COSINES, TANGENTS AND COTANGENTS.

′	0°		1°		2°		3°		4°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.00000	One.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	One.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	One.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	One.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	One.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	One.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	One.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	One.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	One.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	One.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	One.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	.14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	.09671	.99531	.11407	.99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	·99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	84°		83°		82°		81°		80°		

′	10° Sine	10° Cosin	11° Sine	11° Cosin	12° Sine	12° Cosin	13° Sine	13° Cosin	14° Sine	14° Cosin	′
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.93033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95031	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95015	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27812	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.93580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.28241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93379	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	69°		68°		67°		66°		65°		

′	25° Sine	25° Cosin	26° Sine	26° Cosin	27° Sine	27° Cosin	28° Sine	28° Cosin	29° Sine	29° Cosin	′
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	.56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
′	59°		58°		57°		56°		55°		′

′	35°		36°		37°		38°		39°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	40
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	.66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	.72116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	Cosin	Sine	
′	49°		48°		47°		46°		45°		′

′	0°		1°		2°		3°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite.	.01746	57.2900	.03492	28.6363	.05241	19.0811	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		

′	4° Tang	4° Cotang	5° Tang	5° Cotang	6° Tang	6° Cotang	7° Tang	7° Cotang	′
0	.06993	14.3007	.08749	11.4301	.10510	9.51436	.12278	8.14435	60
1	.07022	14.2411	.08778	11.3919	.10540	9.48781	.12308	8.12481	59
2	.07051	14.1821	.08807	11.3540	.10569	9.46141	.12338	8.10536	58
3	.07080	14.1235	.08837	11.3163	.10599	9.43515	.12367	8.08600	57
4	.07110	14.0655	.08866	11.2789	.10628	9.40904	.12397	8.06674	56
5	.07139	14.0079	.08895	11.2417	.10657	9.38307	.12426	8.04756	55
6	.07168	13.9507	.08925	11.2048	.10687	9.35724	.12456	8.02848	54
7	.07197	13.8940	.08954	11.1681	.10716	9.33155	.12485	8.00948	53
8	.07227	13.8378	.08983	11.1316	.10746	9.30599	.12515	7.99058	52
9	.07256	13.7821	.09013	11.0954	.10775	9.28058	.12544	7.97176	51
10	.07285	13.7267	.09042	11.0594	.10805	9.25530	.12574	7.95302	50
11	.07314	13.6719	.09071	11.0237	.10834	9.23016	.12603	7.93438	49
12	.07344	13.6174	.09101	10.9882	.10863	9.20516	.12633	7.91582	48
13	.07373	13.5634	.09130	10.9529	.10893	9.18028	.12662	7.89734	47
14	.07402	13.5098	.09159	10.9178	.10922	9.15554	.12692	7.87895	46
15	.07431	13.4566	.09189	10.8829	.10952	9.13093	.12722	7.86064	45
16	.07461	13.4039	.09218	10.8483	.10981	9.10646	.12751	7.84242	44
17	.07490	13.3515	.09247	10.8139	.11011	9.08211	.12781	7.82428	43
18	.07519	13.2996	.09277	10.7797	.11040	9.05789	.12810	7.80622	42
19	.07548	13.2480	.09306	10.7457	.11070	9.03379	.12840	7.78825	41
20	.07578	13.1969	.09335	10.7119	.11099	9.00983	.12869	7.77035	40
21	.07607	13.1461	.09365	10.6783	.11128	8.98598	.12899	7.75254	39
22	.07636	13.0958	.09394	10.6450	.11158	8.96227	.12929	7.73480	38
23	.07665	13.0458	.09423	10.6118	.11187	8.93867	.12958	7.71715	37
24	.07695	12.9962	.09453	10.5789	.11217	8.91520	.12988	7.69957	36
25	.07724	12.9469	.09482	10.5462	.11246	8.89185	.13017	7.68208	35
26	.07753	12.8981	.09511	10.5136	.11276	8.86862	.13047	7.66466	34
27	.07782	12.8496	.09541	10.4813	.11305	8.84551	.13076	7.64732	33
28	.07812	12.8014	.09570	10.4491	.11335	8.82252	.13106	7.63005	32
29	.07841	12.7536	.09600	10.4172	.11364	8.79964	.13136	7.61287	31
30	.07870	12.7062	.09629	10.3854	.11394	8.77689	.13165	7.59575	30
31	.07899	12.6591	.09658	10.3538	.11423	8.75425	.13195	7.57872	29
32	.07929	12.6124	.09688	10.3224	.11452	8.73172	.13224	7.56176	28
33	.07958	12.5660	.09717	10.2913	.11482	8.70931	.13254	7.54487	27
34	.07987	12.5199	.09746	10.2602	.11511	8.68701	.13284	7.52806	26
35	.08017	12.4742	.09776	10.2294	.11541	8.66482	.13313	7.51132	25
36	.08046	12.4288	.09805	10.1988	.11570	8.64275	.13343	7.49465	24
37	.08075	12.3838	.09834	10.1683	.11600	8.62078	.13372	7.47806	23
38	.08104	12.3390	.09864	10.1381	.11629	8.59893	.13402	7.46154	22
39	.08134	12.2946	.09893	10.1080	.11659	8.57718	.13432	7.44509	21
40	.08163	12.2505	.09923	10.0780	.11688	8.55555	.13461	7.42871	20
41	.08192	12.2067	.09952	10.0483	.11718	8.53402	.13491	7.41240	19
42	.08221	12.1632	.09981	10.0187	.11747	8.51259	.13521	7.39616	18
43	.08251	12.1201	.10011	9.98931	.11777	8.49128	.13550	7.37999	17
44	.08280	12.0772	.10040	9.96007	.11806	8.47007	.13580	7.36389	16
45	.08309	12.0346	.10069	9.93101	.11836	8.44896	.13609	7.34786	15
46	.08339	11.9923	.10099	9.90211	.11865	8.42795	.13639	7.33190	14
47	.08368	11.9504	.10128	9.87338	.11895	8.40705	.13669	7.31600	13
48	.08397	11.9087	.10158	9.84482	.11924	8.38625	.13698	7.30018	12
49	.08427	11.8673	.10187	9.81641	.11954	8.36555	.13728	7.28442	11
50	.08456	11.8262	.10216	9.78817	.11983	8.34496	.13758	7.26873	10
51	.08485	11.7853	.10246	8.76009	.12013	8.32446	.13787	7.25310	9
52	.08514	11.7448	.10275	9.73217	.12042	8.30406	.13817	7.23754	8
53	.08544	11.7045	.10305	9.70441	.12072	8.28376	.13846	7.22204	7
54	.08573	11.6645	.10334	9.67680	.12101	8.26355	.13876	7.20661	6
55	.08602	11.6248	.10363	9.64935	.12131	8.24345	.13906	7.19125	5
56	.08632	11.5853	.10393	9.62205	.12160	8.22344	.13935	7.17594	4
57	.08661	11.5461	.10422	9.59490	.12190	8.20352	.13965	7.16071	3
58	.08690	11.5072	.10452	9.56791	.12219	8.18370	.13995	7.14553	2
59	.08720	11.4685	.10481	9.54106	.12249	8.16398	.14024	7.13042	1
60	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	85°		84°		83°		82°		

′	8° Tang	8° Cotang	9° Tang	9° Cotang	10° Tang	10° Cotang	11° Tang	11° Cotang	′
0	.14054	7.11537	.15838	6.31375	.17633	5.67128	.19438	5.14455	60
1	.14084	7.10038	.15868	6.30189	.17663	5.66165	.19468	5.13658	59
2	.14113	7.08546	.15898	6.29007	.17693	5.65205	.19498	5.12862	58
3	.14143	7.07059	.15928	6.27829	.17723	5.64248	.19529	5.12069	57
4	.14173	7.05579	.15958	6.26655	.17753	5.63295	.19559	5.11279	56
5	.14202	7.04105	.15988	6.25486	.17783	5.62344	.19589	5.10490	55
6	.14232	7.02637	.16017	6.24321	.17813	5.61397	.19619	5.09704	54
7	.14262	6.91174	.16047	6.23160	.17843	5.60452	.19649	5.08921	53
8	.14291	6.99718	.16077	6.22003	.17873	5.59511	.19680	5.08139	52
9	.14321	6.98268	.16107	6.20851	.17903	5.58573	.19710	5.07360	51
10	.14351	6.96823	.16137	6.19703	.17933	5.57638	.19740	5.06584	50
11	.14381	6.95385	.16167	6.18559	.17963	5.56706	.19770	5.05809	49
12	.14410	6.93952	.16196	6.17419	.17993	5.55777	.19801	5.05037	48
13	.14440	6.92525	.16226	6.16283	.18023	5.54851	.19831	5.04267	47
14	.14470	6.91104	.16256	6.15151	.18053	5.53927	.19861	5.03499	46
15	.14499	6.89688	.16286	6.14023	.18083	5.53007	.19891	5.02734	45
16	.14529	6.88278	.16316	6.12899	.18113	5.52090	.19921	5.01971	44
17	.14559	6.86874	.16346	6.11779	.18143	5.51176	.19952	5.01210	43
18	.14588	6.85475	.16376	6.10664	.18173	5.50264	.19982	5.00451	42
19	.14618	6.84082	.16405	6.09552	.18203	5.49356	.20012	4.99695	41
20	.14648	6.82694	.16435	6.08444	.18233	5.48451	.20042	4.98940	40
21	.14678	6.81312	.16465	6.07340	.18263	5.47548	.20073	4.98188	39
22	.14707	6.79936	.16495	6.06240	.18293	5.46648	.20103	4.97438	38
23	.14737	6.78564	.16525	6.05143	.18323	5.45751	.20133	4.96690	37
24	.14767	6.77199	.16555	6.04051	.18353	5.44857	.20164	4.95945	36
25	.14796	6.75838	.16585	6.02962	.18384	5.43966	.20194	4.95201	35
26	.14826	6.74483	.16615	6.01878	.18414	5.43077	.20224	4.94460	34
27	.14856	6.73133	.16645	6.00797	.18444	5.42192	.20254	4.93721	33
28	.14886	6.71789	.16674	5.99720	.18474	5.41309	.20285	4.92984	32
29	.14915	6.70450	.16704	5.98646	.18504	5.40429	.20315	4.92249	31
30	.14945	6.69116	.16734	5.97576	.18534	5.39552	.20345	4.91516	30
31	.14975	6.67787	.16764	5.96510	.18564	5.38677	.20376	4.90785	29
32	.15005	6.66463	.16794	5.95448	.18594	5.37805	.20406	4.90056	28
33	.15034	6.65144	.16824	5.94390	.18624	5.36936	.20436	4.89330	27
34	.15064	6.63831	.16854	5.93335	.18654	5.36070	.20466	4.88605	26
35	.15094	6.62523	.16884	5.92283	.18684	5.35206	.20497	4.87882	25
36	.15124	6.61219	.16914	5.91236	.18714	5.34345	.20527	4.87162	24
37	.15153	6.59921	.16944	5.90191	.18745	5.33487	.20557	4.86444	23
38	.15183	6.58627	.16974	5.89151	.18775	5.32631	.20588	4.85727	22
39	.15213	6.57339	.17004	5.88114	.18805	5.31778	.20618	4.85013	21
40	.15243	6.56055	.17033	5.87080	.18835	5.30928	.20648	4.84300	20
41	.15272	6.54777	.17063	5.86051	.18865	5.30080	.20679	4.83590	19
42	.15302	6.53503	.17093	5.85024	.18895	5.29235	.20709	4.82882	18
43	.15332	6.52234	.17123	5.84001	.18925	5.28393	.20739	4.82175	17
44	.15362	6.50970	.17153	5.82982	.18955	5.27553	.20770	4.81471	16
45	.15391	6.49710	.17183	5.81966	.18986	5.26715	.20800	4.80769	15
46	.15421	6.48456	.17213	5.80953	.19016	5.25880	.20830	4.80068	14
47	.15451	6.47206	.17243	5.79944	.19046	5.25048	.20861	4.79370	13
48	.15481	6.45961	.17273	5.78938	.19076	5.24218	.20891	4.78673	12
49	.15511	6.44720	.17303	5.77936	.19106	5.23391	.20921	4.77978	11
50	.15540	6.43484	.17333	5.76937	.19136	5.22566	.20952	4.77286	10
51	.15570	6.42253	.17363	5.75941	.19166	5.21744	.20982	4.76595	9
52	.15600	6.41026	.17393	5.74949	.19197	5.20925	.21013	4.75906	8
53	.15630	6.39804	.17423	5.73960	.19227	5.20107	.21043	4.75219	7
54	.15660	6.38587	.17453	5.72974	.19257	5.19293	.21073	4.74534	6
55	.15689	6.37374	.17483	5.71992	.19287	5.18480	.21104	4.73851	5
56	.15719	6.36165	.17513	5.71013	.19317	5.17671	.21134	4.73170	4
57	.15749	6.34961	.17543	5.70037	.19347	5.16863	.21164	4.72490	3
58	.15779	6.33761	.17573	5.69064	.19378	5.16058	.21195	4.71813	2
59	.15809	6.32566	.17603	5.68094	.19408	5.15256	.21225	4.71137	1
60	.15838	6.31375	.17633	5.67128	.19438	5.14455	.21256	4.70463	0
′	81° Cotang	81° Tang	80° Cotang	80° Tang	79° Cotang	79° Tang	78° Cotang	78° Tang	′

′	12° Tang	12° Cotang	13° Tang	13° Cotang	14° Tang	14° Cotang	15° Tang	15° Cotang	′
0	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	60
1	.21286	4.69791	.23117	4.32573	.24964	4.00582	.26826	3.72771	59
2	.21316	4.69121	.23148	4.32001	.24995	4.00086	.26857	3.72338	58
3	.21347	4.68452	.23179	4.31430	.25026	3.99592	.26888	3.71907	57
4	.21377	4.67786	.23209	4.30860	.25056	3.99099	.26920	3.71476	56
5	.21408	4.67121	.23240	4.30291	.25087	3.98607	.26951	3.71046	55
6	.21438	4.66458	.23271	4.29724	.25118	3.98117	.26982	3.70616	54
7	.21469	4.65797	.23301	4.29159	.25149	3.97627	.27013	3.70188	53
8	.21499	4.65138	.23332	4.28595	.25180	3.97139	.27044	3.69761	52
9	.21529	4.64480	.23363	4.28032	.25211	3.96651	.27076	3.69335	51
10	.21560	4.63825	.23393	4.27471	.25242	3.96165	.27107	3.68909	50
11	.21590	4.63171	.23424	4.26911	.25273	3.95680	.27138	3.68485	49
12	.21621	4.62518	.23455	4.26352	.25304	3.95196	.27169	3.68061	48
13	.21651	4.61868	.23485	4.25795	.25335	3.94713	.27201	3.67638	47
14	.21682	4.61219	.23516	4.25239	.25366	3.94232	.27232	3.67217	46
15	.21712	4.60572	.23547	4.24685	.25397	3.93751	.27263	3.66796	45
16	.21743	4.59927	.23578	4.24132	.25428	3.93271	.27294	3.66376	44
17	.21773	4.59283	.23608	4.23580	.25459	3.92793	.27326	3.65957	43
18	.21804	4.58641	.23639	4.23030	.25490	3.92316	.27357	3.65538	42
19	.21834	4.58001	.23670	4.22481	.25521	3.91839	.27388	3.65121	41
20	.21864	4.57363	.23700	4.21933	.25552	3.91364	.27419	3.64705	40
21	.21895	4.56726	.23731	4.21387	.25583	3.90890	.27451	3.64289	39
22	.21925	4.56091	.23762	4.20842	.25614	3.90417	.27482	3.63874	38
23	.21956	4.55458	.23793	4.20298	.25645	3.89945	.27513	3.63461	37
24	.21986	4.54826	.23823	4.19756	.25676	3.89474	.27545	3.63048	36
25	.22017	4.54196	.23854	4.19215	.25707	3.89004	.27576	3.62636	35
26	.22047	4.53568	.23885	4.18675	.25738	3.88536	.27607	3.62224	34
27	.22078	4.52941	.23916	4.18137	.25769	3.88068	.27638	3.61814	33
28	.22108	4.52316	.23946	4.17600	.25800	3.87601	.27670	3.61405	32
29	.22139	4.51693	.23977	4.17064	.25831	3.87136	.27701	3.60996	31
30	.22169	4.51071	.24008	4.16530	.25862	3.86671	.27732	3.60588	30
31	.22200	4.50451	.24039	4.15997	.25893	3.86208	.27764	3.60181	29
32	.22231	4.49832	.24069	4.15465	.25924	3.85745	.27795	3.59775	28
33	.22261	4.49215	.24100	4.14934	.25955	3.85284	.27826	3.59370	27
34	.22292	4.48600	.24131	4.14405	.25986	3.84824	.27858	3.58966	26
35	.22322	4.47986	.24162	4.13877	.26017	3.84364	.27889	3.58562	25
36	.22353	4.47374	.24193	4.13350	.26048	3.83906	.27921	3.58160	24
37	.22383	4.46764	.24223	4.12825	.26079	3.83449	.27952	3.57758	23
38	.22414	4.46155	.24254	4.12301	.26110	3.82992	.27983	3.57357	22
39	.22444	4.45548	.24285	4.11778	.26141	3.82537	.28015	3.56957	21
40	.22475	4.44942	.24316	4.11256	.26172	3.82083	.28046	3.56557	20
41	.22505	4.44338	.24347	4.10736	.26203	3.81630	.28077	3.56159	19
42	.22536	4.43735	.24377	4.10216	.26235	3.81177	.28109	3.55761	18
43	.22567	4.43134	.24408	4.09699	.26266	3.80726	.28140	3.55364	17
44	.22597	4.42534	.24439	4.09182	.26297	3.80276	.28172	3.54968	16
45	.22628	4.41936	.24470	4.08666	.26328	3.79827	.28203	3.54573	15
46	.22658	4.41340	.24501	4.08152	.26359	3.79378	.28234	3.54179	14
47	.22689	4.40745	.24532	4.07639	.26390	3.78931	.28266	3.53785	13
48	.22719	4.40152	.24562	4.07127	.26421	3.78485	.28297	3.53393	12
49	.22750	4.39560	.24593	4.06616	.26452	3.78040	.28329	3.53001	11
50	.22781	4.38969	.24624	4.06107	.26483	3.77595	.28360	3.52609	10
51	.22811	4.38381	.24655	4.05599	.26515	3.77152	.28391	3.52219	9
52	.22842	4.37793	.24686	4.05092	.26546	3.76709	.28423	3.51829	8
53	.22872	4.37207	.24717	4.04586	.26577	3.76268	.28454	3.51441	7
54	.22903	4.36623	.24747	4.04081	.26608	3.75828	.28486	3.51053	6
55	.22934	4.36040	.24778	4.03578	.26639	3.75388	.28517	3.50666	5
56	.22964	4.35459	.24809	4.03076	.26670	3.74950	.28549	3.50279	4
57	.22995	4.34879	.24840	4.02574	.26701	3.74512	.28580	3.49894	3
58	.23026	4.34300	.24871	4.02074	.26733	3.74075	.28612	3.49509	2
59	.23056	4.33723	.24902	4.01576	.26764	3.73640	.28643	3.49125	1
60	.23087	4.33148	.24933	4.01078	.26795	3.73205	.28675	3.48741	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	77°		76°		75°		74°		

′	16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	73°		72°		71°		70°		

′	20°		21°		22°		23°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43239	2.31271	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		

′	24°		25°		26°		27°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.44523	2.24604	.46631	2.14451	.48773	2.05030	.50953	1.96261	60
1	.44558	2.24428	.46666	2.14288	.48809	2.04879	.50989	1.96120	59
2	.44593	2.24252	.46702	2.14125	.48845	2.04728	.51026	1.95979	58
3	.44627	2.24077	.46737	2.13963	.48881	2.04577	.51063	1.95838	57
4	.44662	2.23902	.46772	2.13801	.48917	2.04426	.51099	1.95698	56
5	.44697	2.23727	.46808	2.13639	.48953	2.04276	.51136	1.95557	55
6	.44732	2.23553	.46843	2.13477	.48989	2.04125	.51173	1.95417	54
7	.44767	2.23378	.46879	2.13316	.49026	2.03975	.51209	1.95277	53
8	.44802	2.23204	.46914	2.13154	.49062	2.03825	.51246	1.95137	52
9	.44837	2.23030	.46950	2.12993	.49098	2.03675	.51283	1.94997	51
10	.44872	2.22857	.46985	2.12832	.49134	2.03526	.51319	1.94858	50
11	.44907	2.22683	.47021	2.12671	.49170	2.03376	.51356	1.94718	49
12	.44942	2.22510	.47056	2.12511	.49206	2.03227	.51393	1.94579	48
13	.44977	2.22337	.47092	2.12350	.49242	2.03078	.51430	1.94440	47
14	.45012	2.22164	.47128	2.12190	.49278	2.02929	.51467	1.94301	46
15	.45047	2.21992	.47163	2.12030	.49315	2.02780	.51503	1.94162	45
16	.45082	2.21819	.47199	2.11871	.49351	2.02631	.51540	1.94023	44
17	.45117	2.21647	.47234	2.11711	.49387	2.02483	.51577	1.93885	43
18	.45152	2.21475	.47270	2.11552	.49423	2.02335	.51614	1.93746	42
19	.45187	2.21304	.47305	2.11392	.49459	2.02187	.51651	1.93608	41
20	.45222	2.21132	.47341	2.11233	.49495	2.02039	.51688	1.93470	40
21	.45257	2.20961	.47377	2.11075	.49532	2.01891	.51724	1.93332	39
22	.45292	2.20790	.47412	2.10916	.49568	2.01743	.51761	1.93195	38
23	.45327	2.20619	.47448	2.10758	.49604	2.01596	.51798	1.93057	37
24	.45362	2.20449	.47483	2.10600	.49640	2.01449	.51835	1.92920	36
25	.45397	2.20278	.47519	2.10442	.49677	2.01302	.51872	1.92782	35
26	.45432	2.20108	.47555	2.10284	.49713	2.01155	.51909	1.92645	34
27	.45467	2.19938	.47590	2.10126	.49749	2.01008	.51946	1.92508	33
28	.45502	2.19769	.47626	2.09969	.49786	2.00862	.51983	1.92371	32
29	.45538	2.19599	.47662	2.09811	.49822	2.00715	.52020	1.92235	31
30	.45573	2.19430	.47698	2.09654	.49858	2.00569	.52057	1.92098	30
31	.45608	2.19261	.47733	2.09498	.49894	2.00423	.52094	1.91962	29
32	.45643	2.19092	.47769	2.09341	.49931	2.00277	.52131	1.91826	28
33	.45678	2.18923	.47805	2.09184	.49967	2.00131	.52168	1.91690	27
34	.45713	2.18755	.47840	2.09028	.50004	1.99986	.52205	1.91554	26
35	.45748	2.18587	.47876	2.08872	.50040	1.99841	.52242	1.91418	25
36	.45784	2.18419	.47912	2.08716	.50076	1.99695	.52279	1.91282	24
37	.45819	2.18251	.47948	2.08560	.50113	1.99550	.52316	1.91147	23
38	.45854	2.18084	.47984	2.08405	.50149	1.99406	.52353	1.91012	22
39	.45889	2.17916	.48019	2.08250	.50185	1.99261	.52390	1.90876	21
40	.45924	2.17749	.48055	2.08094	.50222	1.99116	.52427	1.90741	20
41	.45960	2.17582	.48091	2.07939	.50258	1.98972	.52464	1.90607	19
42	.45995	2.17416	.48127	2.07785	.50295	1.98828	.52501	1.90472	18
43	.46030	2.17249	.48163	2.07630	.50331	1.98684	.52538	1.90337	17
44	.46065	2.17083	.48198	2.07476	.50368	1.98540	.52575	1.90203	16
45	.46101	2.16917	.48234	2.07321	.50404	1.98396	.52613	1.90069	15
46	.46136	2.16751	.48270	2.07167	.50441	1.98253	.52650	1.89935	14
47	.46171	2.16585	.48306	2.07014	.50477	1.98110	.52687	1.89801	13
48	.46206	2.16420	.48342	2.06860	.50514	1.97966	.52724	1.89667	12
49	.46242	2.16255	.48378	2.06706	.50550	1.97823	.52761	1.89533	11
50	.46277	2.16090	.48414	2.06553	.50587	1.97681	.52798	1.89400	10
51	.46312	2.15925	.48450	2.06400	.50623	1.97538	.52836	1.89266	9
52	.46348	2.15760	.48486	2.06247	.50660	1.97395	.52873	1.89133	8
53	.46383	2.15596	.48521	2.06094	.50696	1.97253	.52910	1.89000	7
54	.46418	2.15432	.48557	2.05942	.50733	1.97111	.52947	1.88867	6
55	.46454	2.15268	.48593	2.05790	.50769	1.96969	.52985	1.88734	5
56	.46489	2.15104	.48629	2.05637	.50806	1.96827	.53022	1.88602	4
57	.46525	2.14940	.48665	2.05485	.50843	1.96685	.53059	1.88469	3
58	.46560	2.14777	.48701	2.05333	.50879	1.96544	.53096	1.88337	2
59	.46595	2.14614	.48737	2.05182	.50916	1.96402	.53134	1.88205	1
60	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	65°		64°		63°		62°		

′	36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	53°		52°		51°		50°		

′	40° Tang	40° Cotang	41° Tang	41° Cotang	42° Tang	42° Cotang	43° Tang	43° Cotang	′
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		

′	44°		′	′	44°		′	′	44°		′
	Tang	Cotang			Tang	Cotang			Tang	Cotang	
0	.96569	1.03553	60	20	.97700	1.02355	40	40	.98843	1.01170	20
1	.96625	1.03493	59	21	.97756	1.02295	39	41	.98901	1.01112	19
2	.96681	1.03433	58	22	.97813	1.02236	38	42	.98958	1.01053	18
3	.96738	1.03372	57	23	.97870	1.02176	37	43	.99016	1.00994	17
4	.96794	1.03312	56	24	.97927	1.02117	36	44	.99073	1.00935	16
5	.96850	1.03252	55	25	.97984	1.02057	35	45	.99131	1.00876	15
6	.96907	1.03192	54	26	.98041	1.01998	34	46	.99189	1.00818	14
7	.96963	1.03132	53	27	.98098	1.01939	33	47	.99247	1.00759	13
8	.97020	1.03072	52	28	.98155	1.01879	32	48	.99304	1.00701	12
9	.97076	1.03012	51	29	.98213	1.01820	31	49	.99362	1.00642	11
10	.97133	1.02952	50	30	.98270	1.01761	30	50	.99420	1.00583	10
11	.97189	1.02892	49	31	.98327	1.01702	29	51	.99478	1.00525	9
12	.97246	1.02832	48	32	.98384	1.01642	28	52	.99536	1.00467	8
13	.97302	1.02772	47	33	.98441	1.01583	27	53	.99594	1.00408	7
14	.97359	1.02713	46	34	.98499	1.01524	26	54	.99652	1.00350	6
15	.97416	1.02653	45	35	.98556	1.01465	25	55	.99710	1.00291	5
16	.97472	1.02593	44	36	.98613	1.01406	24	56	.99768	1.00233	4
17	.97529	1.02533	43	37	.98671	1.01347	23	57	.99826	1.00175	3
18	.97586	1.02474	42	38	.98728	1.01288	22	58	.99884	1.00116	2
19	.97643	1.02414	41	39	.98786	1.01229	21	59	.99942	1.00058	1
20	.97700	1.02355	40	40	.98843	1.01170	20	60	1.00000	1.00000	0
′	Cotang	Tang	′	′	Cotang	Tang	′	′	Cotang	Tang	′
	45°				45°				45°		